МАТЕМАТИЧКИ ВЕСНИК
54 (2002), 13–20

UDK 515.122.2
оригинални научни рад
research paper

WEAKLY θ-CLOSED FUNCTIONS BETWEEN FUZZY TOPOLOGICAL SPACES

Miguel Caldas, Govindappa Navalagi and Ratnesh Saraf

Abstract. In this paper we introduce and characterize fuzzy weakly θ-closed functions between fuzzy topological spaces and also study these functions in relation to some other types of already known functions.

1. Introduction and preliminaries

The concept of fuzzy sets was introduced by L.A. Zadeh in his classical paper [22]. After the discovery of fuzzy subsets, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a modern theory of fuzzy topology is developed. The notion of fuzzy subsets naturally plays a significant role in the study of fuzzy topology which was introduced by C.L. Chang [5]. Ming and Ming [14] introduced the concepts of quasi-coincidence and q-neighbourhoods by which the extensions of functions in fuzzy setting can very interestingly and effectively be carried out. In [18] D.A. Rose defined weakly closed functions in topological spaces. J.H. Park, Y.B. Park and J.S. Park introduced in [17] the notion of weakly closed functions between fuzzy topological spaces. In this paper we introduce and discuss the concept of fuzzy θ-closed function and fuzzy weakly θ-closed function. Also, we obtain several characterizations of these functions comparing with other already existing known functions. Here it is seen that fuzzy θ-closedness implies fuzzy weakly θ-closedness but not conversely. But under a certain condition the converse is also true.

Throughout this paper by (X, τ) or simply by X we mean a fuzzy topological space (fts, shortly) due to Chang [5]. A fuzzy point in X with support $x \in X$ and value p $(0 < p \leqslant 1)$ is denoted by x_p. Two fuzzy sets λ and β are said to be quasi-coincident (q-coincident, shortly), denoted by $\lambda \mathrm{q} \beta$, if there exists $x \in X$ such that $\lambda(x) + \beta(x) > 1$ [14] and by $\bar{\mathrm{q}}$ we denote "is not q-coincident". It is known [14] that $\lambda \leqslant \beta$ if and only if $\lambda \bar{\mathrm{q}} (1 - \beta)$. A fuzzy set λ is said to be q-neighbourhood

AMS Subject Classification: 54 A 40, 54 C 10, 54 D 10
Keywords and phrases: Fuzzy θ-open sets, fuzzy weakly open, fuzzy θ-continuous, fuzzy weakly θ-continuous, fuzzy extremally disconnected spaces, fuzzy almost compact spaces.

(q-nbd) of x_p if there is a fuzzy open set μ such that $x_p q \mu \leqslant \lambda$. The interior, closure and the complement of a fuzzy set $\lambda \in X$ are denoted by $\mathrm{Int}(\lambda)$, $\mathrm{Cl}(\lambda)$ and $1-\lambda=\lambda^c$, respectively.

For definitions and results not explained in this paper, the reader is referred to [1,5,8,9,11,15,17,18] assuming them to be well known.

DEFINITON 1.1. A fuzzy set λ in an fts X is called:
fuzzy preopen [4] if $\lambda \leqslant \mathrm{Int}(\mathrm{Cl}(\lambda))$;
fuzzy preclosed [4] if $\mathrm{Cl}(\mathrm{Int}(\lambda)) \leqslant \lambda$;
fuzzy regular open [2] if $\lambda = \mathrm{Int}(\mathrm{Cl}(\lambda))$;
fuzzy regular closed [2] if $\lambda = \mathrm{Cl}(\mathrm{Int}(\lambda))$;
fuzzy α-open [4] if $\lambda \leqslant \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(\lambda)))$;
fuzzy α-closed [4] if $\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(\lambda))) \leqslant \lambda$.

DEFINITION 1.2. Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be a function from an fts (X,τ_1) into an fts (Y,τ_2). The function f is called:

(i) fuzzy closed [5] if $f(\lambda)$ is a fuzzy closed set of Y, for each fuzzy closed set $\lambda \in X$;

(ii) fuzzy open [5] if $f(\lambda)$ is a fuzzy open set in Y, for each fuzzy open set $\lambda \in X$;

(iii) fuzzy preclosed [19] if $f(\lambda)$ is a fuzzy preclosed set of Y, for each fuzzy closed set $\lambda \in X$.

Recall that a fuzzy point x_p is said to be a fuzzy θ-cluster point of a fuzzy set λ [15], if for every fuzzy open q-nbd μ of x_p, $\mathrm{Cl}(\mu)$ is q-coincident with λ. The set of all fuzzy θ-cluster points of λ is called the fuzzy θ-closure of λ and will be denoted by $\mathrm{Cl}_\theta(\lambda)$. A fuzzy set λ will be called θ-closed if $\lambda = \mathrm{Cl}_\theta(\lambda)$. The complement of a fuzzy θ-closed set is called fuzzy θ-open and the θ-interior of λ denoted by $\mathrm{Int}_\theta(\lambda)$ is defined as $\mathrm{Int}_\theta(\lambda) = \{\, x_p : \text{for some fuzzy open q-nbd, } \beta \text{ of } x_p,\ \mathrm{Cl}(\beta) \leqslant \lambda \,\}$.

LEMMA 1.3. [3] *Let λ be a fuzzy set in an fts X. Then:*

1) λ is fuzzy θ-open if and only if $\lambda = \mathrm{Int}_\theta(\lambda)$;

2) $1 - \mathrm{Int}_\theta(\lambda) = \mathrm{Cl}_\theta(1-\lambda)$ and $\mathrm{Int}_\theta(1-\lambda) = 1 - \mathrm{Cl}_\theta(\lambda)$;

3) $\mathrm{Cl}_\theta(\lambda)$ (resp. $\mathrm{Int}_\theta(\lambda)$) is a fuzzy closed set (resp. fuzzy open set) but not necessarily is a fuzzy θ-closed set (resp. fuzzy θ-open set).

RESULT 1.4. (i) It is easy to see that $\mathrm{Cl}(\lambda) \leqslant \mathrm{Cl}_\theta(\lambda)$ and $\mathrm{Int}_\theta(\lambda) \leqslant \mathrm{Int}(\lambda)$ for any fuzzy set λ in an fts X.

(ii) For a fuzzy open (resp. fuzzy closed) set λ in an fts X, $\mathrm{Cl}(\lambda) = \mathrm{Cl}_\theta(\lambda)$ (resp. $\mathrm{Int}_\theta(\lambda) = \mathrm{Int}(\lambda)$).

DEFINITION 1.5. [8] An fts (X,τ) is fuzzy T_2 (FT_2)-space in the sense of Ganguly and Saha if for two distinct fuzzy points x_λ, y_μ in X:

(i) $x \neq y$ implies that x_λ, y_μ have fuzzy open nbds which are not q-coincident;

(ii) $x = y$ and $\lambda < \mu$ (say) imply that x_λ has a fuzzy open nbd and y_μ has a fuzzy open q-nbd which are not q-coincident.

DEFINITION 1.6. [9] An fts (X, τ) is said to be fuzzy normal if:

(i) for two fuzzy closed sets λ and μ in X with different supports, there exist fuzzy open sets η and γ such that $\lambda \leqslant \eta$, $\mu \leqslant \gamma$ and $\eta \bar{q} \gamma$;

(ii) if $x \in \mathrm{Supp}(\lambda)$ and $x_\mu q \lambda$ then there exist fuzzy open sets η and γ such that $x_\mu q \eta$, $\lambda \leqslant \gamma$ and $\eta \bar{q} \gamma$.

2. Fuzzy weakly θ-closed functions

Now we define a generalized form of weakly closed and closed functions in fuzzy setting.

DEFINITION 2.1. A function $f\colon (X, \tau_1) \to (Y, \tau_2)$ is said to be fuzzy weakly θ-closed if $\mathrm{Cl}_\theta(f(\mathrm{Int}(\beta))) \leqslant f(\beta)$ for each fuzzy closed set β in X.

DEFINITION 2.2. A function $f\colon (X, \tau_1) \to (Y, \tau_2)$ is said to be fuzzy θ-closed if $f(\beta)$ is a fuzzy θ-closed set in Y for each fuzzy closed set β in X.

Clearly, every fuzzy θ-closed function is a fuzzy weakly θ-closed function since $\mathrm{Cl}_\theta(f(\mathrm{Int}(\beta))) \leqslant \mathrm{Cl}_\theta(f(\beta)) = f(\beta)$ for every fuzzy closed subset β of X. Also, every fuzzy θ-closed function is a fuzzy closed function since every fuzzy θ-closed set is a fuzzy closed set, but the converse is not generally true.

EXAMPLE 2.3. Let $X = \{x, y, z\}$ and $Y = \{a, b, c\}$. Fuzzy sets A and B are defined as: $A(x) = 0$, $A(y) = 0.7$, $A(z) = 0.4$; $B(a) = 0$, $B(b) = 0.2$, $B(c) = 0.3$. Let $\tau_1 = \{0, A, 1_X\}$ and $\tau_2 = \{0, B, 1_Y\}$. Then the function $f\colon (X, \tau_1) \to (Y, \tau_2)$ defined by $f(x) = a$, $f(y) = b$ and $f(z) = c$ is fuzzy weakly θ-closed but not fuzzy θ-closed.

EXAMPLE 2.4. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Fuzzy sets A and H are defined as: $A(a) = 0$, $A(b) = 0.2$, $A(c) = 0.7$; $H(x) = 0$, $H(y) = 0.2$, $H(z) = 0.2$. Let $\tau = \{0, A, 1_X\}$ and $\sigma = \{0, H, 1_Y\}$. Then the function $f\colon (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x$, $f(b) = y$ and $f(c) = y$ is fuzzy closed but not fuzzy θ-closed.

THEOREM 2.5. *For a function $f\colon (X, \tau_1) \to (Y, \tau_2)$ the following conditions are equivalent:*

(i) f is fuzzy weakly θ-closed;

(ii) $\mathrm{Cl}_\theta(f(\lambda)) \leqslant f(\mathrm{Cl}(\lambda))$ for every fuzzy open set λ in X.

Proof. *(i)* $\to$ *(ii)*. Let λ be any fuzzy open subset of X. Then $\mathrm{Cl}_\theta(f(\lambda)) = \mathrm{Cl}_\theta(f(\mathrm{Int}(\lambda))) \leqslant \mathrm{Cl}_\theta(f(\mathrm{Int}(\mathrm{Cl}(\lambda)))) \leqslant f(\mathrm{Cl}(\lambda))$.

(ii) $\to$ *(i)*. Let β be any fuzzy closed subset of X. Then, $\mathrm{Cl}_\theta(f(\mathrm{Int}(\beta))) \leqslant f(\mathrm{Cl}(\mathrm{Int}(\beta))) \leqslant f(\mathrm{Cl}(\beta)) = f(\beta)$. ∎

Since the proof of the following theorem is mostly straightforward, it is omitted.

THEOREM 2.6. *For a function $f\colon (X, \tau_1) \to (Y, \tau_2)$ the following conditions are equivalent:*

(i) f is fuzzy weakly θ-closed;

(ii) $\mathrm{Cl}_\theta(f(\lambda)) \leqslant f(\mathrm{Cl}(\lambda))$ *for each fuzzy open set* λ *in* X*;*

(iii) $\mathrm{Cl}_\theta(f(\mathrm{Int}(\beta))) \leqslant f(\beta)$ *for each fuzzy closed subset* β *in* X*;*

(iv) $\mathrm{Cl}_\theta(f(\mathrm{Int}(\beta))) \leqslant f(\beta)$ *for each fuzzy preclosed subset* β *in* X*;*

(v) $\mathrm{Cl}_\theta(f(\mathrm{Int}(\beta))) \leqslant f(\beta)$ *for every fuzzy* α*-closed subset* β *in* X*.*

THEOREM 2.7. *For a function* $f\colon (X,\tau_1) \to (Y,\tau_2)$ *the following conditions are equivalent:*

(i) f *is fuzzy weakly* θ*-closed;*

(ii) $\mathrm{Cl}_\theta(f(\mathrm{Int}(\mathrm{Cl}(\lambda)))) \leqslant f(\mathrm{Cl}(\lambda))$ *for each fuzzy set* λ *in* X*;*

(iii) $\mathrm{Cl}_\theta(f(\mathrm{Int}(\mathrm{Cl}_\theta(\lambda)))) \leqslant f(\mathrm{Cl}_\theta(\lambda))$ *for each fuzzy set* λ *in* X*;*

(iv) $\mathrm{Cl}_\theta(f(\lambda)) \leqslant f(\mathrm{Cl}(\lambda))$ *for each fuzzy preopen set* λ *in* X*.*

Proof. It is clear that $(i) \to (ii) \to (iv) \to (i)$ and $(i) \to (iii)$. For $(iii) \to (iv)$ note that $\mathrm{Cl}_\theta(\lambda) = \mathrm{Cl}(\lambda)$ for each fuzzy preopen subset $\lambda \in X$ [11]. ■

THEOREM 2.8. *If* Y *is a fuzzy regular space, then for a function* $f\colon (X,\tau_1) \to (Y,\tau_2)$ *the following conditions are equivalent:*

(i) f *is fuzzy weakly* θ*-closed;*

(ii) $\mathrm{Cl}_\theta(f(\lambda)) \leqslant f(\mathrm{Cl}(\lambda))$ *for each fuzzy regular open subset* U *of* X*;*

(iii) for each fuzzy subset β *in* Y *and each fuzzy open set* μ *in* X *with* $f^{-1}(\beta) \leqslant \mu$*, there exists a fuzzy* θ*-open set* δ *in* Y *with* $\beta \leqslant \delta$ *and* $f^{-1}(\delta) \leqslant \mathrm{Cl}(\mu)$*;*

(iv) for each fuzzy point y_p *in* Y *and each fuzzy open set* μ *in* X *with* $f^{-1}(y_p) \leqslant \mu$*, there exists a fuzzy* θ*-open set* δ *in* Y *containing* y_p*, and* $f^{-1}(\delta) \leqslant \mathrm{Cl}(\mu)$*.*

Proof. It is clear that $(i) \to (ii)$ and $(iii) \to (iv)$. To show that $(ii) \to (iii)$, let β be a fuzzy subset in Y and let μ be fuzzy open in X with $f^{-1}(\beta) \leqslant \mu$. Then $f^{-1}(\beta)\bar{\mathrm{q}}\,\mathrm{Cl}(1_X - \mathrm{Cl}(\mu))$ and consequently $\beta\bar{\mathrm{q}}f(\mathrm{Cl}(1_X - \mathrm{Cl}(\mu)))$. Since $1_X - \mathrm{Cl}(\mu)$ is fuzzy regular open, $\beta\bar{\mathrm{q}}\,\mathrm{Cl}_\theta(f(1_X - \mathrm{Cl}(\mu)))$ by (ii). Let $\delta = 1_Y - \mathrm{Cl}_\theta(f(1_X - \mathrm{Cl}(\mu)))$. Then by Corollary 3.6 of [15] δ is fuzzy θ-open with $\beta \leqslant \delta$ and $f^{-1}(\delta) \leqslant 1_X - f^{-1}(\mathrm{Cl}_\theta(f(1_X - \mathrm{Cl}(\mu)))) \leqslant 1_X - f^{-1}f(1_X - \mathrm{Cl}(\mu)) \leqslant \mathrm{Cl}(\mu)$.

$(iv) \to (i)$. Let β be fuzzy closed in X and let $y_p \in 1_Y - f(\beta)$. Since $f^{-1}(y_p) \leqslant 1_X - \beta$, there exists a fuzzy θ-open δ in Y with $y_p \in \delta$ and $f^{-1}(\delta) \leqslant \mathrm{Cl}(1_X - \beta) = 1_X - \mathrm{Int}(\beta)$ by (iv). Therefore $\delta\bar{\mathrm{q}}f(\mathrm{Int}(\beta))$, so that $y_p \in 1_Y - \mathrm{Cl}_\theta(f(\mathrm{Int}(\beta)))$. Thus $(iv) \to (i)$. ■

Note that the fact of Y being fuzzy regular space in Theorem 2.8 is only used in $(ii) \to (iii)$.

THEOREM 2.9. *If* $f\colon (X,\tau_1) \to (Y,\tau_2)$ *is fuzzy weakly* θ*-closed, then for each fuzzy point* y_p *in* Y *and each fuzzy open q-nbd* μ *of* $f^{-1}(y_p)$ *in* X*, there exists a fuzzy* θ*-open q-nbd* δ *of* y_p *in* Y *such that* $f^{-1}(\delta) \leqslant \mathrm{Cl}(\mu)$*.*

Proof. Let μ be any fuzzy open q-nbd of $f^{-1}(y_p)$ in X. Then $\mu(x) + p > 1$ and hence there exists a positive real number α such that $\mu(x) > \alpha > 1 - p$. Then μ is a fuzzy open q-nbd of $f^{-1}(y_\alpha)$. By Theorem 2.8(iv) there exists a fuzzy θ-open set δ containing y_α in Y such that $f^{-1}(\delta) \leqslant \mathrm{Cl}(\mu)$. Now, $\delta(y) > \alpha$ and hence $\delta(y) > 1 - p$. Thus δ is a fuzzy θ-open q-nbd of y_p. ■

Next we investigate conditions under which fuzzy weakly θ-closed functions are fuzzy θ-closed.

THEOREM 2.10. *If $f\colon (X,\tau_1) \to (Y,\tau_2)$ is fuzzy weakly θ-closed and if for each fuzzy closed subset β of X and each fiber $f^{-1}(y_p) \leqslant 1_X - \beta$ there exists a fuzzy open q-nbd μ in X such that $f^{-1}(y_p) \leqslant \mu \leqslant \mathrm{Cl}(\mu) \leqslant 1_X - \beta$. Then f is fuzzy θ-closed.*

Proof. Let β be any fuzzy closed subset of X and let $y_p \in 1_Y - f(\beta)$. Then $f^{-1}(y_p)\bar{\mathrm{q}}\beta$ and hence $f^{-1}(y_p) \leqslant 1_X - \beta$. By hypothesis, there exists a fuzzy open q-nbd μ in X such that $f^{-1}(y_p) \leqslant \mu \leqslant \mathrm{Cl}(\mu) \leqslant 1_X - \beta$. Since f is fuzzy weakly θ-closed, by Theorem 2.9, there exists a fuzzy θ-open q-nbd ν in Y with $y_p \in \nu$ and $f^{-1}(\nu) \leqslant \mathrm{Cl}(\mu)$. Therefore, we obtain $f^{-1}(\nu)\bar{\mathrm{q}}\beta$ and hence $\nu\bar{\mathrm{q}}f(\beta)$, this shows that $y_p \notin \mathrm{Cl}_\theta(f(\beta))$. Therefore, $f(\beta)$ is fuzzy θ-closed in Y and f is fuzzy θ-closed. ■

Now we give the following

DEFINITION 2.11. Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be a function from an fts (X,τ_1) into an fts (Y,τ_2). The function f is called:

(i) fuzzy contra-closed if $f(\lambda)$ is a fuzzy open subset of Y for each fuzzy closed set $\lambda \in X$.

(ii) fuzzy contra-θ-open if $f(\lambda)$ is a fuzzy θ-closed subset of Y for each fuzzy open set $\lambda \in X$.

THEOREM 2.12. *(i) If $f\colon (X,\tau_1) \to (Y,\tau_2)$ is fuzzy preclosed and fuzzy contra-closed, then f is fuzzy weakly θ-closed.*

(ii) If $f\colon (X,\tau_1) \to (Y,\tau_2)$ is fuzzy contra-θ-open, then f is fuzzy weakly θ-closed.

Proof. (i) Let β be a fuzzy closed subset of X. Since f is fuzzy preclosed, $\mathrm{Cl}_\theta(\mathrm{Int}(f(\beta))) = \mathrm{Cl}(\mathrm{Int}(f(\beta))) \leqslant f(\beta)$ and since f is fuzzy contra-closed $f(\beta)$ is fuzzy open. Therefore $\mathrm{Cl}_\theta(f(\mathrm{Int}(\beta))) \leqslant \mathrm{Cl}_\theta(f(\beta)) = \mathrm{Cl}_\theta(\mathrm{Int}(f(\beta))) \leqslant f(\beta)$.

(ii) Let β be a fuzzy closed subset of X. Then, $\mathrm{Cl}_\theta(f(\mathrm{Int}(\beta))) = f(\mathrm{Int}(\beta)) \leqslant f(\beta)$. ■

THEOREM 2.13. *If Y is a fuzzy regular space and if $f\colon (X,\tau_1) \to (Y,\tau_2)$ is a fuzzy weakly θ-closed function, then for every fuzzy subset β in Y and every fuzzy open set λ in X with $f^{-1}(\beta) \leqslant \lambda$, there exists a fuzzy θ-closed set δ in Y such that $\beta \leqslant \delta$ and $f^{-1}(\delta) \leqslant \mathrm{Cl}(\lambda)$.*

Proof. Let β be a fuzzy subset of Y and let λ be a fuzzy open subset of X with $f^{-1}(\beta) \leqslant \lambda$. Put $\delta = \mathrm{Cl}_\theta(f(\mathrm{Int}(\mathrm{Cl}(\lambda))))$. By Corollary 3.6 of [15] δ is a fuzzy θ-closed set in Y such that $\beta \leqslant \delta$ since $\beta \leqslant f(\lambda) \leqslant f(\mathrm{Int}(\mathrm{Cl}(\lambda))) \leqslant \mathrm{Cl}_\theta(f(\mathrm{Int}(\mathrm{Cl}(\lambda)))) = \delta$. And since f is fuzzy weakly θ-closed, $f^{-1}(\delta) \leqslant \mathrm{Cl}(\lambda)$. ■

COROLLARY 2.14. *Let Y be a fuzzy regular space. If $f\colon (X,\tau_1) \to (Y,\tau_2)$ is fuzzy weakly θ-closed, then for every fuzzy point y_p in Y and every fuzzy open set λ*

in X with $f^{-1}(y_p) \leqslant \lambda$, there exists a fuzzy θ-closed set δ in Y containing y_p such that $f^{-1}(\delta) \leqslant \mathrm{Cl}(\lambda)$.

A fuzzy set β in an fts X is fuzzy θ-compact if for each cover Ω of β by fuzzy open q-nbd μ in X, there is a finite family $\{\mu_1, \ldots, \mu_n\}$ in Ω such that $\beta \leqslant \mathrm{Int}(\bigcup\{\mathrm{Cl}(\mu_i) : i = 1, 2, \ldots, n\})$.

THEOREM 2.15. *If $f\colon (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly θ-closed with all fibers fuzzy θ-closed, then $f(\beta)$ is fuzzy θ-closed for each fuzzy θ-compact β in X.*

Proof. Let β be fuzzy θ-compact and let $y_p \in 1_Y - f(\beta)$. Then $f^{-1}(y_p)\bar{\mathrm{q}}\beta$. Since $f^{-1}(y_p)$ is fuzzy θ-closed, then for each $x_p \in \beta$ there is a fuzzy open q-nbd μ_{x_p} containing x_p in X and $\mathrm{Cl}(\mu_{x_p})\bar{\mathrm{q}}f^{-1}(y_p)$. Clearly $\Omega = \{\mu_{x_p} : x_p \in \beta\}$ is a fuzzy open q-nbd cover of β and since β is fuzzy θ-compact, there is a finite family $\{\mu_{x_1}, \ldots, \mu_{x_n}\}$ in Ω such that $\beta \leqslant \mathrm{Int}(\lambda)$, where $\lambda = \bigcup\{\mathrm{Cl}(\mu_{x_i}) : i = 1, \ldots, n\}$. Since f is fuzzy weakly θ-closed, by Theorem 2.9 there exists a fuzzy θ-open q-nbd δ in Y containing y_p and $f^{-1}(\delta) \leqslant \mathrm{Cl}(1_X - \lambda) = 1_X - \mathrm{Int}(\lambda) \leqslant 1_X - \beta$. Therefore $\delta\bar{\mathrm{q}}f(\beta)$. Thus $y_p \in 1_Y - \mathrm{Cl}_\theta(f(\beta))$. This shows that $f(\beta)$ is fuzzy θ-closed. ∎

Two non-empty fuzzy subsets λ and β in X are strongly fuzzy separated [1] if there exist fuzzy open sets μ and ν in X with $\lambda \leqslant \mu$ and $\beta \leqslant \nu$ and $\mathrm{Cl}(\mu)\bar{\mathrm{q}}\,\mathrm{Cl}(\nu)$. If λ and β are fuzzy singleton sets we may speak of fuzzy points being strongly fuzzy separated. We will use the fact that in a fuzzy normal space [9], disjoint fuzzy closed sets are strongly fuzzy separated.

THEOREM 2.16. *If $f\colon (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly θ-closed surjection and all pairs of disjoint fibers are strongly fuzzy separated, then Y is fuzzy T_2.*

Proof. Let y_p and y_q be two fuzzy points in Y. Let μ and ν be fuzzy open sets in X such that $f^{-1}(y_p) \leqslant \mu$ and $f^{-1}(y_q) \leqslant \nu$ with $\mathrm{Cl}(\mu)\bar{\mathrm{q}}\,\mathrm{Cl}(\nu)$. By fuzzy weak θ-closedness (Theorem 2.8(iv)) there are fuzzy θ-open sets λ and β in Y such that $y_p \leqslant \lambda$ and $y_q \leqslant \beta$, $f^{-1}(\lambda) \leqslant \mathrm{Cl}(\mu)$ and $f^{-1}(\beta) \leqslant \mathrm{Cl}(\nu)$. Therefore $\lambda\bar{\mathrm{q}}\beta$, because $\mathrm{Cl}(\mu)\bar{\mathrm{q}}\,\mathrm{Cl}(\nu)$ and f is surjective. Then Y is fuzzy T_2. ∎

COROLLARY 2.17. *If $f\colon (X, \tau_1) \to (Y, \tau_2)$ is weakly fuzzy θ-closed surjection with all fuzzy closed fibers and X is fuzzy normal, then Y is fuzzy T_2.*

DEFINITION 2.18. A family $\{\lambda_\alpha : \alpha \in \Omega\}$ of fuzzy open subsets of a fuzzy topological space (X, τ) is an open cover if $\bigcup\{\lambda_\alpha : \alpha \in \Omega\} = X$. An fts X is said to be fuzzy almost compact [12,6] (resp. fuzzy c-compact) if every fuzzy open cover (resp. fuzzy closed cover) contains a finite subfamily $\{\lambda_{\alpha_i} : i = 1, 2, \ldots, n\}$ such that $X = \bigcup_{i=1}^{n} \mathrm{Cl}(\lambda_{\alpha_i})$. A fuzzy subset λ of X is fuzzy almost compact relative to X (resp. fuzzy c-compact relative to X) if every cover of λ by fuzzy open (resp. fuzzy closed) sets of X has a finite subfamily whose fuzzy closures cover X.

Recall that an fts (X, τ) is said to be fuzzy extremally disconnected if the closure of every fuzzy open set of X is fuzzy open in X [11].

LEMMA 2.19. [21] *A function $f\colon (X, \tau_1) \to (Y, \tau_2)$ is fuzzy open if and only if for each fuzzy subset β in Y, $f^{-1}(\mathrm{Cl}(\beta)) \leqslant \mathrm{Cl}(f^{-1}(\beta))$.*

THEOREM 2.20. *Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be a fuzzy θ-open and fuzzy weakly θ-closed function from a fuzzy extremally disconnected space X into a fuzzy regular space Y such that $f^{-1}(y_p)$ is fuzzy almost compact relative to X for each fuzzy point y_p in Y. If λ is fuzzy c-compact relative to Y, then $f^{-1}(\lambda)$ is fuzzy almost compact.*

Proof. Let $\{\nu_\beta : \beta \in I\}$ be a fuzzy open cover of $f^{-1}(\lambda)$. Then for each $y_p \in \lambda \cap f(X)$, $f^{-1}(y_p) \leqslant \bigcup\{\mathrm{Cl}(\nu_\beta) : \beta \in I(y_p)\} = \delta_{y_p}$ for some finite subfamily $I(y_p)$ of I. Since X is fuzzy extremally disconnected, each $\mathrm{Cl}(\nu_\beta)$ is fuzzy open, hence δ_{y_p} is fuzzy open in X. So by Corollary 2.14, there exists a fuzzy θ-closed set μ_{y_p} containing y_p such that $f^{-1}(\mu_{y_p}) \leqslant \mathrm{Cl}(\delta_{y_p})$. Then, $\{\mu_{y_p} : y_p \in \lambda \cap f(X)\} \cup \{1_Y - f(X)\}$ is a fuzzy θ-closed cover of λ, $\lambda \leqslant \bigcup\{\mathrm{Cl}(\mu_{y_p}) : y_p \in K\} \cup \{\mathrm{Cl}(1_Y - f(X))\}$ for some finite fuzzy subset K of $\lambda \cap f(X)$. Hence by Lemma 2.19, $f^{-1}(\lambda) \leqslant \bigcup\{f^{-1}(\mathrm{Cl}(\mu_{y_p})) : y_p \in K\} \cup \{f^{-1}(\mathrm{Cl}(1_Y - f(X)))\} \leqslant \bigcup\{\mathrm{Cl}(f^{-1}(\mu_{y_p})) : y_p \in K\} \cup \{\mathrm{Cl}(f^{-1}(1_Y - f(X)))\} \leqslant \{\mathrm{Cl}(f^{-1}(\mu_{y_p})) : y_p \in K\}$, so $f^{-1}(\lambda) \leqslant \bigcup\{\mathrm{Cl}(\nu_\beta) : \beta \in I(y_p), y_p \in K\}$. Therefore $f^{-1}(\lambda)$ is fuzzy almost compact. ■

COROLLARY 2.21. *Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be as in Theorem 2.20. If Y is c-compact, then X is almost compact.*

REFERENCES

[**1**] N. Ajmal and B.K. Tyagi, *Regular fuzzy spaces and fuzzy almost regular spaces*, Mat. Vesnik **40** (1988), 97–108.

[**2**] K.K. Azad, *On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. Appl. **82** (1981), 14–32.

[**3**] R.N. Bhoumik and A. Mukherjee, *Fuzzy weakly completely continuous functions*, Fuzzy Sets and Systems **55** (1993), 347–354.

[**4**] A.S. Bin Shahna, *On fuzzy strong semicontinuity and fuzzy precontinuity*, Fuzzy Sets and Systems **44** (1991), 303–308.

[**5**] C.L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182–192.

[**6**] A. Di Concilio and G. Gerla, *Almost compact sets in fuzzy topological spaces*, Fuzzy Sets and Systems **13** (1984), 187–192

[**7**] S. Ganguly and S. Saha, *A note on semi-open sets in fuzzy topological spaces*, Fuzzy Sets and Systems **18** (1986), 83–96.

[**8**] S. Ganguly and S. Saha, *On separation axioms and T_i-continuity*, Fuzzy Sets and Systems, 265–275.

[**9**] S. Ganguly and S. Saha, *A note on compactness in a fuzzy setting*, Fuzzy Sets and Systems **34** (1990), 117–124.

[**10**] B. Ghosh, *Semi-continuous and semi-closed mappings and semi-connectedness in fuzzy setting*, Fuzzy Sets and Systems **35** (1990), 345–355.

[**11**] B. Ghosh, *Fuzzy extremally disconnected spaces*, Fuzzy Sets and Systems **46** (1992), 245–250.

[**12**] A. Haydar Es, *Almost compactness and near compactness in fuzzy topological spaces*, Fuzzy Sets and Systems **22** (1987), 289–295.

[**13**] A.S. Mashhour, M.H. Ghanim and M.A. Fath Alla, *On fuzzy non-continuous mappings*, Bull. Cal. Math. Soc. **78** (1986), 57–69.

[**14**] P.P. Ming and L.Y. Ming, *Fuzzy topology I. Neighborhood structure of fuzzy point and Moore-Smith convergence*, J. Math. Anal. Appl. **76** (1980), 571–594.

[**15**] M.N. Mukherjee and S.P. Sinha, *Fuzzy θ-closure operator on fuzzy topological spaces*, Internat. J. Math. & Math. Sci. **14** (1991), 309–314.

[**16**] M.N. Mukherjee and S.P. Sinha, *On some weaker forms of fuzzy continuous and fuzzy open mappings on fuzzy topological spaces*, Fuzzy Sets and Systems **32** (1989), 103–114.

[**17**] J.H. Park, Y.B. Park and J.S. Park, *Fuzzy weakly open mappings and fuzzy RS-compact sets*, Far East J. Math. Sci., Special vol. (1997), Part II, 201–212.

[**18**] D.A. Rose, *On weak openness and almost openness*, Internat. J. Math. & Math. Sci. **7** (1984), 35–40.

[**19**] M.K. Singal and N. Prakash, *Fuzzy preopen sets and fuzzy preseparation axioms*, Fuzzy Sets and Systems **44** (1991), 273–281.

[**20**] T.H. Yalvaç, *Semi-interior and semi-closure of a fuzzy set*, J. Math. Anal. Appl. **132** (1988), 356–364.

[**21**] T.H. Yalvaç, *Fuzzy sets and functions on fuzzy spaces*, J. Math. Anal. Appl. **126** (1987), 409–423.

[**22**] L.A. Zadeh, *Fuzzy sets*, Inform. and Control **8** (1965), 338–353.

(received 20.10.2001)

M. Caldas, Departamento de Matematica Aplicada, Universidade Federal Fluminense, Rua Mario Santos Braga, s/n, CEP: 24020-140, Niteroi, RJ Brasil

E-mail: gmamccs@vm.uff.br

G. B. Navalagi, Department of Mathematics, KLE Societys, G. H. College, Haveri–581110, Karnataka, India

E-mail: gnavalagi@hotmail.com

R. K. Saraf, Department of Mathematics, Govt. K.N.G. College, Damoh–470661, M.P., India