МАТЕМАТИЧКИ ВЕСНИК
60 (2008), 225–232

UDK 517.546
оригинални научни рад
research paper

ON THE UNIQUENESS OF MEROMORPHIC FUNCTIONS SHARING THREE WEIGHTED VALUES

Indrajit Lahiri and Gautam Kumar Ghosh

Abstract. We prove a uniqueness theorem for meromorphic functions sharing three weighted values, as consequences of which a number of results follow.

1. Introduction, Definitions and Results

Let f and g be two non-constant meromorphic functions defined in the open complex plane $\mathbb{C}$. For $a \in \mathbb{C} \cup \{\infty\}$ we say that f, g share the value a CM (counting multiplicities) if f, g have the same a-points with the same multiplicity and we say that f, g share the value a IM (ignoring multiplicities) if we do not consider the multiplicities. We denote by E a set of nonnegative real numbers of finite linear measure, not necessarily the same at each of its occurrence. By $S(r,f)$ we mean any quantity satisfying $S(r,f) = o\{T(r,f)\}$ as $r \to \infty (r \notin E)$, where $T(r,f)$ is the Nevanlinna characteristic function of f. By $\overline{N}_0(r,a;f,g)$ we denote the reduced counting function of the common a-points of f and g. If f and g share $0, 1, \infty$ IM, we denote by $N_0(r)$ the counting function of those zeros of $f-g$ which are not the zeros of $f(f-1)$ and $1/f$. For the standard notations and definitions of the value distribution theory we refer the reader to [1].

Let a_1, a_2, a_3, a_4 be four elements of $\mathbb{C}$. Then the cross-ratio of these numbers is defined as

$$(a_1, a_2, a_3, a_4) = \frac{(a_1 - a_3)(a_2 - a_4)}{(a_2 - a_3)(a_1 - a_4)}.$$

If $a_k = \infty$ for some $k \in \{1, 2, 3, 4\}$ then we define the cross-ratio as

$$(a_1, a_2, a_3, a_4) = \lim_{a_k \to \infty} \frac{(a_1 - a_3)(a_2 - a_4)}{(a_2 - a_3)(a_1 - a_4)}.$$

If a, b, c, d are distinct elements of $\mathbb{C} \cup \{\infty\}$, in the paper we denote by $L(w)$ the following bilinear transformation

$$L(w) = \frac{(w-c)(b-d)}{(w-d)(b-c)}.$$

Clearly $L(a) = (a, b, c, d)$ and $a = (a, 1, 0, \infty)$.

AMS Subject Classification: 30D35
Keywords and phrases: Meromorphic function, weighted sharing, uniqueness.

In 1989 N. Terglane [6] proved the following theorem.

THEOREM A. *Let a, b, c, d be four distinct elements of $\mathbb{C}\cup\{\infty\}$. Further suppose that f and g are two distinct nonconstant meromorphic functions sharing b, c, d CM. If $\overline{N}_0(r, a; f, g) \neq S(r, f)$ and $(a, b, c, d) \in \{-1, 2, \frac{1}{2}\}$ then f is a bilinear transformation of g.*

In 2002 H. X. Yi and X. M. Li [7] considered the problem of removing the hypothesis $(a, b, c, d) \in \{-1, 2, \frac{1}{2}\}$ in Theorem A. They proved the following results.

THEOREM B. *Let a_1, a_2, b, c, d be five distinct elements of $\mathbb{C} \cup \{\infty\}$ and f, g be two nonconstant meromorphic functions sharing b, c, d CM. If $\overline{N}_0(r, a_1; f, g) \neq S(r, f)$ and $\overline{N}_0(r, a_2; f, g) \neq S(r, f)$ then $f \equiv g$.*

THEOREM C. *Let f and g be two distinct nonconstant meromorphic functions sharing three values $b, c, d \in \mathbb{C} \cup \{\infty\}$ CM. If $\overline{N}_0(r, a; f, g) \neq S(r, f)$ for some $a \in \mathbb{C} \cup \{\infty\} \setminus \{b, c, d\}$, then (a, b, c, d) is a rational number and $\overline{N}(r, a; f) = T(r, f)+S(r, f)$, $\overline{N}(r, a; g) = T(r, g)+S(r, g)$ and $\overline{N}_0(r, a; f, g) = \frac{1}{p}T(r, f)+S(r, f)$, where p is a positive integer.*

THEOREM D. *Let f and g be two distinct nonconstant meromorphic functions sharing three values $b, c, d \in \mathbb{C} \cup \{\infty\}$ CM. If $\overline{N}_0(r, a; f, g) = T(r, f) + S(r, f)$ for some $a \in \mathbb{C} \cup \{\infty\} \setminus \{b, c, d\}$, then $(a, b, c, d) \in \{-1, 2, \frac{1}{2}\}$, f is a bilinear transformation of g and assume one of the following relations:*

(I) $L(f) = e^{\gamma}$, $L(g) = e^{-\gamma}$ *and* $(a, b, c, d) = -1$,

(II) $L(f) = 1 + e^{\gamma}$, $L(g) = 1 + e^{-\gamma}$ *and* $(a, b, c, d) = 2$,

(III) $L(f) = \dfrac{1}{1 + e^{\gamma}}$, $L(g) = \dfrac{1}{1 + e^{-\gamma}}$ *and* $(a, b, c, d) = \dfrac{1}{2}$,

where γ is a nonconstant entire function.

THEOREM E. *Let f and g be two distinct nonconstant meromorphic functions sharing three values $b, c, d \in \mathbb{C} \cup \{\infty\}$ CM. If $\overline{N}_0(r, a; f, g) \neq S(r, f)$ and $\overline{N}_0(r, a; f, g) \neq T(r, f) + S(r, f)$ for some $a \in \mathbb{C} \cup \{\infty\} \setminus \{b, c, d\}$, then $(a, b, c, d)(\neq 0, 1, -1, 2, \frac{1}{2})$ is a rational number and*

$$\overline{N}_0(r, a; f, g) = \frac{1}{p}T(r, f) + S(r, f),$$

where $p(> 1)$ is a positive integer, f is not a bilinear transformation of g and assumes one of the following relations:

(I) $L(f) = \dfrac{e^{s\gamma} - 1}{e^{(p+1)\gamma} - 1}$, $L(g) = \dfrac{e^{-s\gamma} - 1}{e^{-(p+1)\gamma} - 1}$ *and* $(a, b, c, d) = \dfrac{s}{p + 1}$,

(II) $L(f) = \dfrac{e^{(p+1)\gamma} - 1}{e^{(p+1-s)\gamma} - 1}$, $L(g) = \dfrac{e^{-(p+1)\gamma} - 1}{e^{-(p+1-s)\gamma} - 1}$ *and* $(a, b, c, d) = \dfrac{p + 1}{p + 1 - s}$,

(III) $L(f) = \dfrac{e^{s\gamma} - 1}{e^{-(p+1-s)\gamma} - 1}$, $L(g) = \dfrac{e^{-s\gamma} - 1}{e^{(p+1-s)\gamma} - 1}$ *and* $(a, b, c, d) = \dfrac{s}{s - p - 1}$,

where γ is a nonconstant entire function and $1 \leq s \leq p$.

Considering the following examples we see that in Theorem D the strength of sharing of none of the values can be weakened from CM to IM.

EXAMPLE 1.1. Let $f = \dfrac{4(1-e^z)}{1-3e^z}$ and $g = \dfrac{(1-e^z)^3}{1-3e^z}$. Then f and g share 0 IM and $1, \infty$ CM. Since $f - 2 = \dfrac{2(1+e^z)}{1-3e^z}$ and $g - 2 = \dfrac{(1+e^z)(4e^z - e^{2z} - 1)}{1-3e^z}$, we see that $\overline{N}_0(r, 2; f, g) = T(r, f) + S(r, f)$ but the conclusion of Theorem D does not hold.

EXAMPLE 1.2. Let $f = \dfrac{e^z - 3}{1-3e^z}$ and $g = \dfrac{e^{2z}(e^z - 3)}{1-3e^z}$. Then f , g share $0, \infty$ CM and 1 IM. Since $f + 1 = -\dfrac{2(1+e^z)}{1-3e^z}$ and $g + 1 = \dfrac{(1+e^z)(1+e^{2z} - 4e^z)}{1-3e^z}$, we see that $\overline{N}_0(r, -1; f, g) = T(r, f) + S(r, f)$ but the conclusion of Theorem D does not hold.

EXAMPLE 1.3. Let $f = \dfrac{1-3e^z}{4(1-e^z)}$ and $g = \dfrac{1-3e^z}{(1-e^z)^3}$. Then f, g share $0, 1$ CM and ∞ IM. Since $f - \dfrac{1}{2} = -\dfrac{1+e^z}{4(1-e^z)}$ and $g - \dfrac{1}{2} = \dfrac{(1+e^z)(1-4e^z+e^{2z})}{2(1-e^z)^3}$, we see that $\overline{N}_0(r, \frac{1}{2}; f, g) = T(r, f) + S(r, f)$ but the conclusion of Theorem D does not hold.

Interchanging f and g in the above examples we see that in Theorem E also CM sharing of values cannot be replaced by IM sharing of values.

In the paper we investigate the possibility of relaxing the nature of sharing the three values. To this end we use the idea of weighted value sharing which measures how close a value is being shared CM or being shared IM. We also intend to prove a single uniqueness theorem which includes all the results mentioned above.

In the following definition we explain the idea of weighted value sharing.

DEFINITION 1.1. [2, 3] Let k be a nonnegative integer or infinity. For $a \in \mathbb{C} \cup \{\infty\}$, we denote by $E_k(a; f)$ the set of all a-points of f where an a-point of multiplicity m is counted m times if $m \le k$ and $k+1$ times if $m > k$. If $E_k(a; f) = E_k(a; g)$, we say that f and g share the value a with weight k.

The definition implies that if f and g share a value a with weight k then z_0 is a zero of $f - a$ with multiplicity $m(\le k)$ if and only if it is a zero of $g - a$ with multiplicity $m(\le k)$ and z_0 is a zero of $f - a$ with multiplicity $m(> k)$ if and only if it is a zero of $g - a$ with multiplicity $n(> k)$ where m is not necessarily equal to n.

We write f, g share (a, k) to mean that f, g share the value a with weight k. Clearly if f, g share (a, k) then f, g share (a, p) for all integers p, $0 \le p < k$. Also we note that f, g share a value a IM or CM if and only if f, g share $(a, 0)$ or (a, ∞) respectively.

We now state the main results of the paper.

THEOREM 1.1. *Let f and g be two distinct nonconstant meromorphic functions sharing* $(0, k_1)$, $(1, k_2)$, (∞, k_3), *where* $k_1 k_2 k_3 > k_1 + k_2 + k_3 + 2$. *If* $\overline{N}_0(r, a; f, g) \ne$

$S(r,f)$ for some $a \notin \{0,1,\infty\}$ then one of the following holds:

(I) $f = \dfrac{e^{s\gamma}-1}{e^{(p+1)\gamma}-1}$, $g = \dfrac{e^{-s\gamma}-1}{e^{-(p+1)\gamma}-1}$ *and* $a = \dfrac{s}{p+1}$,

(II) $f = \dfrac{e^{(p+1)\gamma}-1}{e^{(p+1-s)\gamma}-1}$, $g = \dfrac{e^{-(p+1)\gamma}-1}{e^{-(p+1-s)\gamma}-1}$ *and* $a = \dfrac{p+1}{p+1-s}$,

(III) $f = \dfrac{e^{s\gamma}-1}{e^{-(p+1-s)\gamma}-1}$, $g = \dfrac{e^{-s\gamma}-1}{e^{(p+1-s)\gamma}-1}$ *and* $a = \dfrac{s}{s-p-1}$,

where s and p are positive integers with $1 \le s \le p$ and s, $p+1$ are relatively prime and γ is a nonconstant entire function. Further

$$\overline{N}_0(r,a;f,g) = \frac{1}{p}T(r,f) + S(r,f).$$

Theorem 1.2. *Let f and g be two distinct nonconstant meromorphic functions sharing (b,k_1), (c,k_2), (d,k_3), where $k_1k_2k_3 > k_1+k_2+k_3+2$ and b,c,d are three distinct elements of $\mathbb{C}\cup\{\infty\}$. If $\overline{N}_0(r,a;f,g) \ne S(r,f)$ for some $a \in \mathbb{C}\cup\{\infty\}\setminus\{b,c,d\}$ then one of the following holds:*

(I) $L(f) = \dfrac{e^{s\gamma}-1}{e^{(p+1)\gamma}-1}$, $L(g) = \dfrac{e^{-s\gamma}-1}{e^{-(p+1)\gamma}-1}$ *and* $(a,b,c,d) = \dfrac{s}{p+1}$,

(II) $L(f) = \dfrac{e^{(p+1)\gamma}-1}{e^{(p+1-s)\gamma}-1}$, $L(g) = \dfrac{e^{-(p+1)\gamma}-1}{e^{-(p+1-s)\gamma}-1}$ *and* $(a,b,c,d) = \dfrac{p+1}{p+1-s}$,

(III) $L(f) = \dfrac{e^{s\gamma}-1}{e^{-(p+1-s)\gamma}-1}$, $L(g) = \dfrac{e^{-s\gamma}-1}{e^{(p+1-s)\gamma}-1}$ *and* $(a,b,c,d) = \dfrac{s}{s-p-1}$,

where s and p are positive integers with $1 \le s \le p$ and s, $p+1$ are relatively prime and γ is a nonconstant entire function. Further

$$\overline{N}_0(r,a;f,g) = \frac{1}{p}T(r,f) + S(r,f).$$

Following corollaries of Theorem 1.2 improve Theorem A–Theorem E respectively.

Corollary 1.1. *Theorem A holds even if f, g share (b,k_1), (c,k_2), (d,k_3), where $k_1k_2k_3 > k_1+k_2+k_3+2$.*

Corollary 1.2. *Theorem B holds even if f, g share (b,k_1), (c,k_2), (d,k_3), where $k_1k_2k_3 > k_1+k_2+k_3+2$.*

Corollary 1.3. *Theorem C holds even if f, g share (b,k_1), (c,k_2), (d,k_3), where $k_1k_2k_3 > k_1+k_2+k_3+2$.*

Corollary 1.4. *Theorem D holds even if f, g share (b,k_1), (c,k_2), (d,k_3), where $k_1k_2k_3 > k_1+k_2+k_3+2$.*

Corollary 1.5. *Theorem E holds even if f, g share (b,k_1), (c,k_2), (d,k_3), where $k_1k_2k_3 > k_1+k_2+k_3+2$.*

2. Lemmas

In this section we state three lemmas, which are necessary to prove the main results.

Lemma 2.1. [4] *Let f and g be two distinct non-constant meromorphic functions sharing $(0,k_1)$, $(1,k_2)$, and (∞,k_3), where $k_j (j=1,2,3)$ are positive integers satisfying $k_1k_2k_3 > k_1+k_2+k_3+2$. If $\limsup\limits_{r\to\infty, r\notin E} \dfrac{N_0(r)}{T(r,f)} > \dfrac{1}{2}$ then one of the following relations holds: (i) $f+g\equiv 1$, (ii) $(f-1)(g-1)\equiv 1$, and (iii) $fg\equiv 1$.*

Lemma 2.2. [4] *Let f and g be two distinct non-constant meromorphic functions sharing $(0,k_1)$, $(1,k_2)$ and (∞,k_3), where $k_j (j=1,2,3)$ are positive integers satisfying $k_1k_2k_3 > k_1+k_2+k_3+2$. If $0 < \limsup\limits_{r\to\infty, r\notin E} \dfrac{N_0(r)}{T(r,f)} \le \dfrac{1}{2}$ then one of the following holds:*

(i) $f = \dfrac{e^{s\gamma}-1}{e^{(p+1)\gamma}-1}$, $g = \dfrac{e^{-s\gamma}-1}{e^{-(p+1)\gamma}-1}$ $(1\le s\le p)$,

(ii) $f = \dfrac{e^{(p+1)\gamma}-1}{e^{(p+1-s)\gamma}-1}$, $g = \dfrac{e^{-(p+1)\gamma}-1}{e^{-(p+1-s)\gamma}-1}$ $(1\le s\le p)$,

(iii) $f = \dfrac{e^{s\gamma}-1}{e^{-(p+1-s)\gamma}-1}$, $g = \dfrac{e^{-s\gamma}-1}{e^{(p+1-s)\gamma}-1}$ $(1\le s\le p)$,

where s and $p (\ge 2)$ are positive integers such that s, $p+1$ are relatively prime and γ is a nonconstant entire function.

Lemma 2.3. [5] *Let f be a non-constant meromorphic function and*

$$R(f) = \frac{\sum_{i=0}^{m} a_i f^i}{\sum_{j=0}^{n} b_j f^j}$$

be a non-constant irreducible rational function in f with constant coefficients $\{a_i\}$ and $\{b_j\}$ satisfying $a_m \ne 0$ and $b_n \ne 0$. Then

$$T(r,R(f)) = \max\{m,n\}T(r,f) + O(1).$$

3. Proofs of theorems and corollaries

Proof of Theorem 1.1. Since $\overline{N}_0(r,a;f,g) \le N_0(r)$ and $\overline{N}_0(r,a;f,g) \ne S(r,f)$, we see that

$$\limsup_{r\to\infty, r\notin E} \frac{N_0(r)}{T(r,f)} > 0.$$

We now consider the following two cases.

Case I. Let $\limsup_{r\to\infty, r\notin E} \dfrac{N_0(r)}{T(r,f)} > \dfrac{1}{2}$. Then the three possibilities of Lemma 2.1 come up for consideration.

Let $f+g\equiv 1$. Since $\overline{N}_0(r,a;f,g) \neq S(r,f)$, we see that $a=\frac{1}{2}$. Also 0 and 1 are Picard's exceptional values of f and g. So there exists a nonconstant entire function γ such that $f=\dfrac{1}{e^\gamma+1}$ and $g=\dfrac{1}{e^{-\gamma}+1}$, which is possibility (I) of the theorem for $p=1$.

Let $(f-1)(g-1)\equiv 1$. Since $\overline{N}_0(r,a;f,g) \neq S(r,f)$, we see that $a=2$. Also 1 and ∞ are Picard's exceptional values of f and g. So there exists a nonconstant entire function γ such that $f=e^\gamma+1$ and $g=e^{-\gamma}+1$, which is possibility (II) of the theorem for $p=1$.

Let $fg\equiv 1$. Since $\overline{N}_0(r,a;f,g) \neq S(r,f)$, we see that $a=-1$. Also 0 and ∞ are Picard's exceptional values of f and g. So there exists a nonconstant entire function γ such that $f=-e^\gamma$ and $g=-e^{-\gamma}$, which is possibility (III) of the theorem for $p=1$.

Also we see in view of Lemma 2.3 that

$$\overline{N}_0(r,a;f,g)=\overline{N}(r,1;e^\gamma)=T(r,e^\gamma)+S(r,e^\gamma)=T(r,f)+S(r,f).$$

Case II. Let $0<\limsup_{r\to\infty, r\notin E} \dfrac{N_0(r)}{T(r,f)} \leq \dfrac{1}{2}$. Then we consider the three possibilities of Lemma 2.2.

Let

$$f=\frac{e^{s\gamma}-1}{e^{(p+1)\gamma}-1}=\frac{1+e^\gamma+e^{2\gamma}+\cdots+e^{(s-1)\gamma}}{1+e^\gamma+e^{2\gamma}+\cdots+e^{p\gamma}}$$

and

$$g=\frac{e^{-s\gamma}-1}{e^{-(p+1)\gamma}-1}=\frac{1+e^{-\gamma}+e^{-2\gamma}+\cdots+e^{-(s-1)\gamma}}{1+e^{-\gamma}+e^{-2\gamma}+\cdots+e^{-p\gamma}}.$$

Since $\overline{N}_0(r,a;f,g) \neq S(r,f)$, we see from above that $a=\frac{s}{p+1}$, which is attained at the roots of $e^\gamma-1=0$. This is possibility (I) of the theorem.

Let

$$f=\frac{e^{(p+1)\gamma}-1}{e^{(p+1-s)\gamma}-1}=\frac{1+e^\gamma+e^{2\gamma}+\cdots+e^{p\gamma}}{1+e^\gamma+e^{2\gamma}+\cdots+e^{(p-s)\gamma}}$$

and

$$g=\frac{e^{-(p+1)\gamma}-1}{e^{-(p+1-s)\gamma}-1}=\frac{1+e^{-\gamma}+e^{-2\gamma}+\cdots+e^{-p\gamma}}{1+e^{-\gamma}+e^{-2\gamma}+\cdots+e^{-(p-s)\gamma}}.$$

Since $\overline{N}_0(r,a;f,g) \neq S(r,f)$, we see from above that $a=\frac{p+1}{p+1-s}$, which is attained at the roots of $e^\gamma-1=0$. This is possibility (II) of the theorem.

Let

$$f=\frac{e^{s\gamma}-1}{e^{-(p+1-s)\gamma}-1}=-e^\gamma\frac{1+e^\gamma+e^{2\gamma}+\cdots+e^{(s-1)\gamma}}{1+e^{-\gamma}+e^{-2\gamma}+\cdots+e^{-(p-s)\gamma}}$$

and

$$g = \frac{e^{-s\gamma} - 1}{e^{(p+1-s)\gamma} - 1} = -e^{\gamma} \frac{1 + e^{-\gamma} + e^{-2\gamma} + \cdots + e^{-(s-1)\gamma}}{1 + e^{\gamma} + e^{2\gamma} + \cdots + e^{(p-s)\gamma}}.$$

Since $\overline{N}_0(r, a; f, g) \neq S(r, f)$, we see from above that $a = \frac{s}{s-p-1}$, which is attained at the roots of $e^{\gamma} - 1 = 0$. This is possibility (III) of the theorem.

Again from above we see in view of Lemma 2.3

$$\overline{N}_0(r, a; f, g) = \overline{N}(r, 1; e^{\gamma}) = T(r, e^{\gamma}) + S(r, e^{\gamma}) = \frac{1}{p} T(r, f) + S(r, f).$$

This proves the theorem. ∎

Proof of Theorem 1.2. From the hypotheses of the theorem we see that $L(f)$ and $L(g)$ share $(0, k_2)$, $(1, k_1)$, (∞, k_3). Also in view of Lemma 2.3 we get

$$\overline{N}_0(r, L(a); L(f), L(g)) = N_0(r, a; f, g) \neq S(r, f) = S(r, L(f)).$$

Since k_1 and k_2 are interchangeable, Theorem 1.2 follows from Theorem 1.1 applied to $L(f)$ and $L(g)$. This proves the theorem. ∎

Proof of Corollary 1.1. From the hypotheses we see that the possibilities of Theorem 1.2 occur only for $p = 1$. Hence $L(f)$ is a bilinear transformation of $L(g)$. Therefore f is a bilinear transformation of g. ∎

Proof of Corollary 1.2. If $f \not\equiv g$, by Theorem 1.2 we see that $L(a_1) = L(a_2)$ and so $a_1 = a_2$, which contradicts the hypotheses. Hence $f \equiv g$. ∎

Proof of Corollary 1.3. By the hypotheses and Theorem 1.2 we see that $L(a) = (a, b, c, d)$ is a rational number.

Also in view of Theorem 1.2 and Lemma 2.3 we can verify that

$$\overline{N}(r, a; f) = \overline{N}(r, L(a); L(f)) = T(r, L(f)) + S(r, L(f)) = T(r, f) + S(r, f)$$

and

$$\overline{N}(r, a; g) = \overline{N}(r, L(a); L(g)) = T(r, L(g)) + S(r, L(g)) = T(r, g) + S(r, g).$$

Further by Theorem 1.2 we obtain

$$\overline{N}_0(r, a; f, g) = \frac{1}{p} T(r, f) + S(r, f). \quad ∎$$

Proof of Corollary 1.4. By the hypotheses we see that $p = 1$ in Theorem 1.2 and so the corollary follows. ∎

Proof of Corollary 1.5. By the hypotheses we get $p > 1$ in Theorem 1.2. Hence the corollary follows. ∎

REFERENCES

[**1**] W. K. Hayman, *Meromorphic Functions*, The Clarendon Press, Oxford 1964.

[**2**] I. Lahiri, *Weighted sharing and uniqueness of meromorphic functions*, Nagoya Math. J., **161** (2001), 193–206.

[**3**] I. Lahiri, *Characteristic functions of meromorphic functions sharing three values with finite weights*, Complex Variable Theory Appl., **50**, 1 (2005), 69–78.

[**4**] I. Lahiri and P. Sahoo, *Uniqueness of meromorphic functions sharing three weighted values*, Bull. Malaysian Math. Sci. Soc., (2) **31** (1) (2008), 67–75.

[**5**] A. Z. Mohonko, *On the Nevanlinna characteristic of some meromorphic functions*, Funct. Anal. Appl., **14** (1971), 83–87.

[**6**] N. Terglane, *Identitätssätze in* **C** *Meromorpher Funktionen als Ergebnis von Werteteilung*, Diplomarbeit, RWTH Aachen (1989).

[**7**] H. X. Yi and X. M. Li, *On a result of Terglane concerning uniqueness of meromorphic functions*, Indian J. Pure Appl. Math., **33**, 12 (2002), 1785–1798.

(received 26.09.2007, in revised form 14.11.2007)

Department of Mathematics, University of Kalyani, West Bengal 741235, India.

E-mail: indr9431@dataone.in

Department of Mathematics, G.M.S.M. Mahavidyalaya, Bireswarpur, South 24 Parganas, India.

E-mail: g80g@rediffmail.com