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ON (f,g)-DERIVATIONS OF B-ALGEBRAS

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Abstract. In this paper, as a generalization of derivation of a B-algebra, we introduce the notion of f-derivation and (f,g)-derivation of a B-algebra. Also, some properties of (f,g)-derivation of commutative B-algebra are investigated.

1. Introduction and preliminaries

Y. Imai and K. Iséki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras [7, 8]. It is known that the class of BCK-algebras is a proper subclass of the class BCI-algebras. In [5, 6], Q. P. Hu and X. Li introduced a wide class of abstract algebras, BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of BCH-algebras. In [9], Y. B. Jun, E. H. Roh and H.S. Kim introduced the notion of BH-algebras, which is a generalization of $BCH/BCI/BCK$-algebras. Recently, J. Neggers and H. S. Kim introduced in [12] a new notion, called a B-algebra. This class of algebras is related to several classes of interest such as $BCH/BCI/BCK$-algebras. In [1], N. O. Al-Shehrie introduced the notion of derivation in B-algebras which is defined in a way similar to the notion in ring theory (see [2, 3, 10, 15]) and investigated some properties related to this concept.

In this paper, we introduce the notions of f-derivation and (f,g)-derivation of a B-algebra and some related are explored. Also, using the concept of derivation of commutative B-algebra we investigate some of its properties.

We recall the notion of a B-algebra and review some properties which we will need in the next section.

A *B-algebra* [12] is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following conditions, for all $x,y,z \in X$: $(B1)$ $x*x=0$; $(B2)$ $x*0=x$; $(B3)$ $(x*y)*z = x*(z*(0*y))$. A B-algebra $(X,*,0)$ is said to be *commutative* [12] if $x*(0*y) = y*(0*x)$, for all $x,y \in X$.

In any B-algebra X, the following properties are valid, for all $x,y,z \in X$ [4, 12]: (1) $(x*y)*(0*y) = x$; (2) $x*(y*z) = (x*(0*z))*y$; (3) $x*y=0$ implies

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that $x = y$; (4) $0 * (0 * x) = x$; (5) $(x * z) * (y * z) = x * y$; (6) $0 * (x * y) = y * x$; (7) $x * z = y * z$ implies that $x = y$ (right cancelation law); (8) $z * x = z * y$ implies that $x = y$ (left cancelation law). Moreover, if X is a commutative B-algebra, according to [11] we have: (9) $(0 * x) * (0 * y) = y * x$; (10) $(z * y) * (z * x) = x * y$; (11) $(x*y)*z = (x*z)*y$; (12) $(x*(x*y))*y = 0$; (13) $(x*z)*(y*t) = (t*z)*(y*x)$. For a B-algebra X, one can define binary operation "$\wedge$" as $x \wedge y = y * (y * x)$, for all $x, y \in X$. If $(X, *, 0)$ is a commutative B-algebra, then by (12) and (3), we get $y * (y * x) = x$, for all $x, y \in X$ that means $x \wedge y = x$.

A mapping f of a B-algebra X in to itself is called an *endomorphism* of X if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$. Note that $f(0) = 0$.

Let $(X, *, +, 0)$ be an algebra of type $(2, 2, 0)$ satisfying $B1, B2, B3$ and $B4:$ $x + y = x * (0 * y)$, for all $x, y \in X$. Then, $(X, *, 0)$ is a B-algebra. Conversely, if $(X, *, 0)$ be a B-algebra and we define $x + y$ by $x * (0 * y)$, for all $x, y \in X$, then $(X, *, +, 0)$ obeys the equations $B1 - B4$ (see [15]).

2. (f, g)-derivation of B-algebras

In this section, we introduce the notion of f-derivation and (f, g)-derivation of B-algebras.

Definition 1. [1] Let X be a B-algebra. By a *left-right derivation* (briefly, (l, r)-*derivation*) of X, a self map d of X satisfying the identity $d(x*y) = (d(x)*y) \wedge (x*d(y))$, for all $x, y \in X$. If d satisfies the identity $d(x*y) = (x*d(y)) \wedge (d(x)*y)$, for all $x, y \in X$, then it is said that d is a *right-left derivation* (briefly, (r, l)-*derivation*) of X. Moreover, if d is both an (l, r)- and (r, l)-derivation, it is said that d is a *derivation*.

Definition 2. [1] A self map d of a B-algebra X is said to be *regular* if $d(0) = 0$. If $d(0) \neq 0$, then d is called an *irregular* map.

Definition 3. Let X be a B-algebra. A *left-right f-derivation* (briefly, (l, r)-*f-derivation*) of X is a self map d of X satisfying the identity $d(x * y) = (d(x) * f(y)) \wedge (f(x) * d(y))$, for all $x, y \in X$, where f is an endomorphism of X. If d satisfies the identity $d(x * y) = (f(x) * d(y)) \wedge (d(x) * f(y))$, for all $x, y \in X$, then we say d is a *right-left f-derivation* (briefly, (r, l)-*f-derivation*) of X. Moreover, if d is both an (l, r)- and (r, l)-f-derivation, we say d is an *f-derivation*.

Example 1. Let $X = \{0, 1, 2\}$ and the binary operation $*$ is defined as follows:

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then, $(X,*,0)$ is a B-algebra (see [12]). Define the map $d,f : X \longrightarrow X$ by

$$d(x) = f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 2 & \text{if } x = 1 \\ 1 & \text{if } x = 2. \end{cases}$$

Then, f is an endomorphism. It is easily to check that d is both (l,r)- and (r,l)-f-derivation of X. So d is an f-derivation. Now, we define $d' = 0$. Then, d' is not an (l,r)-f-derivation, since $d'(1*2) = 0$ but $(d'(1)*f(2)) \wedge (f(1)*d'(2)) = (0*1) \wedge (2*0) = 2$. Also, d' is not an (r,l)-f-derivation, since $d'(1*2) = 0$ but $(f(1)*d'(2)) \wedge (d'(1)*f(2)) = (2*0) \wedge (0*1) = 2$.

Theorem 1. *Let d be an (l,r)-f-derivation of B-algebra X. Then, $d(0) = d(x)*f(x)$, for all $x \in X$.*

Proof. For all $x \in X$, we have:

$$\begin{aligned} d(0) &= d(x*x) = (d(x)*f(x)) \wedge (f(x)*d(x)) \\ &= (f(x)*d(x))*((f(x)*d(x))*(d(x)*f(x))) \\ &= ((f(x)*d(x))*(0*(d(x)*f(x))))*(f(x)*d(x)) \\ &= ((f(x)*d(x))*(f(x)*d(x)))*(f(x)*d(x)) \\ &= 0*(f(x)*d(x)) = d(x)*f(x). \quad \blacksquare \end{aligned}$$

Theorem 2. *Let d be an (r,l)-f-derivation of B-algebra X. Then, $d(0) = f(x)*d(x)$ and $d(x) = d(x) \wedge f(x)$, for all $x \in X$.*

Proof. For all $x \in X$, we have:

$$\begin{aligned} d(0) &= d(x*x) = (f(x)*d(x)) \wedge (d(x)*f(x)) \\ &= (d(x)*f(x))*((d(x)*f(x))*(f(x)*d(x))) \\ &= ((d(x)*f(x))*(0*(f(x)*d(x))))*(d(x)*f(x)) \\ &= ((d(x)*f(x))*(d(x)*f(x)))*(d(x)*f(x)) \\ &= 0*(d(x)*f(x)) = f(x)*d(x). \end{aligned}$$

Also, we have for all $x \in X$,

$$\begin{aligned} d(x)*0 = d(x) &= d(x*0) = (f(x)*d(0)) \wedge (d(x)*f(0)) \\ &= d(x)*(d(x)*(f(x)*d(0))) = d(x)*(d(x)*(f(x)*(f(x)*d(x)))). \end{aligned}$$

By (8) and (3), we get $d(x) \wedge f(x) = d(x)$. $\blacksquare$

Corollary 1. *Let d be an (l,r)-f-derivation ((r,l)-f-derivation) of B-algebra X. Then,* (1) *d is injective if and only if f be injective;* (2) *If d is regular, then $d = f$;* (3) *If there is an element $x_0 \in X$ such that $d(x_0) = f(x_0)$, then $d = f$.*

Proof. Let d be an (l,r)-f-derivation.

(1) Suppose that d is injective and $f(x) = f(y)$, $x, y \in X$. Then, $d(0) = d(x) * f(x)$ and $d(0) = d(y) * f(y)$, by Theorem 1. So, $d(x) * f(x) = d(y) * f(y)$. Thus, $d(x) = d(y)$, by (7). Therefore, $x = y$, since d is injective.

Conversely, suppose that f is injective and $d(x) = d(y)$, $x, y \in X$. Then, $d(0) = d(x)*f(x)$ and $d(0) = d(y)*f(y)$, by Theorem 1. So, $d(x)*f(x) = d(y)*f(y)$. Thus $f(x) = f(y)$, by (8). Therefore $x = y$, since f is injective.

(2) Suppose that d is regular and $x \in X$. Then $0 = d(0) = d(x) * f(x)$, by Theorem 1. Hence, $d(x) = f(x)$, by (3).

(3) Suppose that there is an element $x_0 \in X$ such that $d(x_0) = f(x_0)$. Then, $d(x_0) * f(x_0) = 0$. So, $d(0) = 0$, by Theorem 1. Part (2) implies that $d = f$.

Similarly, when d is an (r, l)-f-derivation, the proof follows by Theorem 2. ■

Definition 4. Let X be a B-algebra. A *left-right* (f, g)*-derivation* (briefly, (l, r)-(f, g)*-derivation*) of X is a self map d of X satisfying the identity $d(x * y) = (d(x) * f(y)) \wedge (g(x) * d(y))$, for all $x, y \in X$, where f, g are endomorphisms of X. If d satisfies the identity $d(x*y) = (f(x)*d(y)) \wedge (d(x)*g(y))$, for all $x, y \in X$, then we say d is a *right-left* (f, g)*-derivation* (briefly, (r, l)-(f, g)*-derivation*) of X. Moreover, if d is both an (l, r)- and (r, l)-(f, g)-derivation, then d is a (f, g)*-derivation*.

It is clear that if the function g is equal to the function f, then the (f, g)-derivation is f-derivation defined in Definition 3. Also, if we choose the functions f and g the identity functions, then the (f, g)-derivation that we define coincides with the derivation defined in Definition 1.

Example 2. Let $(X, *, 0)$, d and f are as Example 1. Define $g = I$, where I is an identity function. It is easily checked that d is an (f, g)-derivation. But d is not an (l, r)-(g, f)-derivation, since $d(1 * 2) = 1$ but $(d(1) * g(2)) \wedge (f(1) * d(2)) = (2 * 2) \wedge (2 * 1) = 0$. Also, d is not an (r, l)-(g, f)-derivation, since $d(1 * 2) = 1$ but $(g(1) * d(2)) \wedge (d(1) * f(2)) = (1 * 1) \wedge (2 * 1) = 0$.

Example 3. Let $X = \{0, 1, 2, 3\}$ and binary operation $*$ is defined as:

$*$	0	1	2	3
0	0	2	1	3
1	1	0	3	2
2	2	3	0	1
3	3	1	2	0

Then, $(X, *, 0)$ is a B-algebra (see [1]). Define maps $d, f, g : X \longrightarrow X$ by

$$d(x) = f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 2 & \text{if } x = 1 \\ 1 & \text{if } x = 2 \\ 3 & \text{if } x = 3 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 0 & \text{if } x = 0 \\ 3 & \text{if } x = 1, 2, 3. \end{cases}$$

Then, f and g are endomorphisms. It is easily to check that d is an (f, g)-derivation.

Theorem 3. *Let d be a self map of a B-algebra X. Then, the following hold:*

(1) If d is a regular (l, r)-(f, g)-derivation of X, then $d(x) = d(x) \wedge g(x)$, for all $x \in X$;

(2) If d is an (r, l)-(f, g)-derivation of X, then $d(x) = f(x) \wedge d(x)$, for all $x \in X$ if and only if d is a regular.

Proof. (1) Suppose that d is a regular (l, r)-(f, g)-derivation of X and $x \in X$. Then, $d(x) = d(x * 0) = (d(x) * f(0)) \wedge (g(x) * d(0)) = d(x) \wedge g(x)$.

(2) Suppose that d is an (r, l)-(f, g)-derivation of X. If $d(x) = f(x) \wedge d(x)$, for all $x \in X$, then $d(0) = f(0) \wedge d(0) = d(0) * (d(0) * 0) = d(0) * d(0) = 0$. Conversely, suppose that $d(0) = 0$. Then, $d(x) = d(x * 0) = (f(x) * d(0)) \wedge (d(x) * g(0)) = f(x) \wedge d(x)$, for all $x \in X$. ■

Now, we investigate (f, g)-derivation of commutative B-algebras.

Theorem 4. *Let X be a commutative B-algebra. Then, for all $x, y \in X$,*

(1) If d is an (l, r)-(f, g)-derivation of X, then $d(x*y) = d(x)*f(y)$. Moreover, $d(0) = d(x) * f(x)$;

(2) If d is an (r, l)-(f, g)-derivation of X, then $d(x*y) = f(x)*d(y)$. Moreover $d(0) = f(x) * d(x)$;

(3) If d is an (l, r)-(f, g)-derivation ((r, l)-(f, g)-derivation), then Corollary 1 is valid.

Proof. The proof is clear. ■

Theorem 5. *Let X be a commutative B-algebra and f, g be endomorphisms. If $d = f$, then d is an (f, g)-derivation.*

Proof. Suppose that $x, y \in X$. Then, $d(x * y) = f(x * y) = f(x) * f(y) = d(x) * f(y) = (d(x) * f(y)) \wedge (g(x) * d(y))$. So, d is an (r, l)-(f, g)-derivation. Also, we have $d(x*y) = f(x*y) = f(x)*f(y) = f(x)*d(y) = (f(x)*d(y)) \wedge (d(x)*g(y))$. Hence, d is an (l, r)-(f, g)-derivation. Therefore, d is an (f, g)-derivation. ■

Theorem 6. *Let d be an (l, r)-(f, g)-derivation ((r, l)-(f, g)-derivation) of a commutative B-algebra X. Then, for all $x, y \in X$, (1) $d(x) = d(0) + f(x)$; (2) $d(x + y) = d(x) + d(y) - d(0)$; (3) $d(x) * d(y) = f(x) * f(y)$.*

Proof. (1) Suppose that d is an (l, r)-(f, g)-derivation of X and $x \in X$. Then, by Theorem 4(1), we have $d(x) = d(0*(0*x)) = d(0)*f(0*x) = d(0)*(0*f(x)) = d(0) + f(x)$. Now, suppose that d is an (r, l)-(f, g)-derivation of X and $x \in X$. Then, by Theorem 4(2), we get $d(x) = d(x*0) = f(x)*d(0)$ and $d(0) = f(0)*d(0)$. So, $d(x) = f(x) * (0 * d(0)) = d(0) * (0 * f(x)) = d(0) + f(x)$.

(2) By (1), for all $x, y \in X$, $d(x + y) = d(0) + f(x + y) = d(0) + f(x) + d(0) + f(y) - d(0) = d(x) + d(y) - d(0)$.

(3) Suppose that d is an (l, r)-(f, g)-derivation of X. Then, by Theorem 4, $d(x) * f(x) = d(0) = d(y) * f(y)$, for all $x, y \in X$. Thus, $(d(y) * f(y)) * (d(x) *$

$f(x)) = 0$. Now, by (13) we obtain $(f(x) * f(y)) * (d(x) * d(y)) = 0$. Therefore, $d(x) * d(y) = f(x) * f(y)$.

When d is an (r, l)-(f, g)-derivation, the proof is similar. ∎

Let $(X, *, 0)$ be an B-algebra and $x \in X$. Define $x^n := x^{n-1} * (0 * x)$ $(n \geq 1)$ and $x^0 := 0$. Note that $x^1 = x^0 * (0 * x) = 0 * (0 * x) = x$ [12].

THEOREM 7. [12] *Let $(X, *, 0)$ be a B-algebra. Then, for all $x \in X$,*

$$x^m * x^n = \begin{cases} x^{m-n} & \text{if } m \geq n \\ 0 * x^{n-m} & \text{if } m < n. \end{cases}$$

THEOREM 8. *Let $(X, *, 0)$ be a commutative B-algebra X. For all $x \in X$,*

(1) *If d be an (l, r)-(f, g)-derivation, then*

$$d(x^m * x^n) = \begin{cases} d(0) * (0 * f(x))^{m-n} & \text{if } m \geq n \\ d(0) * f(x)^{n-m} & \text{if } m < n. \end{cases}$$

(2) *If d be an (r, l)-(f, g)-derivation, then*

$$d(x^m * x^n) = \begin{cases} f(x)^{m-n-1} * (0 * d(x)) & \text{if } m \geq n \\ (0 * d(x)) * f(x)^{n-m-1} & \text{if } m < n. \end{cases}$$

Proof. It is clear that $(x * y^n) * y = x * y^{n+1}$, for all $x, y \in X$ and $n \geq 1$. So by induction we get, $d(x^n) = d(0) * (0 * f(x))^n$, where d is an (l, r)-(f, g)-derivation. Also, we have $d(x^n) = f(x)^{n-1} * (0 * d(x))$, where d is an (r, l)-(f, g)-derivation. Now (1) and (2) follow by Theorem 7. ∎

THEOREM 9. *Let X be a commutative B-algebra and f, g be endomorphisms such that $f \circ f = f$. Also, let d and d' be (l, r)-(f, g)-derivations ((r, l)-(f, g)-derivations) of X. Then, $d \circ d'$ is also an (l, r)-(f, g)-derivation ((r, l)-(f, g)-derivation) of X.*

Proof. Let d and d' are the (l, r)-(f, g)-derivations of X. Then, by Theorem 4, for all $x, y \in X$, we have $(d \circ d')(x * y) = d(d'(x) * f(y)) = d(d'(x)) * f(f(y)) = d \circ d'(x) * f(y) = (d \circ d'(x) * f(y)) \wedge (g(x) * d \circ d'(y))$. Thus, $d \circ d'$ is a (l, r)-(f, g)-derivation of X. Now, suppose that d, d' are (r, l)-(f, g)-derivations of X. Similarly, we can prove $d \circ d'$ is a (r, l)-(f, g)-derivation of X. ∎

THEOREM 10. *Let X be a commutative B-algebra, d and d' be (f, g)-derivations of X such that $f \circ d = d \circ f$, $d' \circ f = f \circ d'$. Then, $d \circ d' = d' \circ d$.*

Proof. Since d' is an (l, r)-(f, g)-derivation and d is an (r, l)-(f, g)-derivation of X, for all $x, y \in X$,

$$\begin{aligned} (d \circ d')(x * y) &= d((d'(x) * f(y)) \wedge (g(x) * d'(y))) \\ &= d(d'(x) * f(y)) = f \circ d'(x) * d \circ f(y). \end{aligned} \tag{1}$$

Also, since d' is an (l,r)-(f,g)-derivation and d is an (r,l)-(f,g)-derivation of X, for all $x, y \in X$, we have

$$\begin{aligned}(d' \circ d)(x * y) &= d'((f(x) * d(y)) \wedge (d(x) * g(y))) = d'(f(x) * d(y)) \\ &= d' \circ f(x) * f \circ d(y) = f \circ d'(x) * d \circ f(y). \qquad (2)\end{aligned}$$

By the relations (1) and (2), we have $d \circ d'(x * y) = d' \circ d(x * y)$, for all $x, y \in X$. By putting $y = 0$, we get $d \circ d'(x) = d' \circ d(x)$, for all $x \in X$. ■

Let X be a B-algebra and d, d' be two self maps of X. We define $d \bullet d' : X \longrightarrow X$ as follows: $(d \bullet d')(x) = d(x) * d'(x)$, for all $x \in X$.

THEOREM 11. *Let X be a commutative B-algebra and d, d' be (f,g)-derivations of X. Then, (1) $(f \circ d') \bullet (d \circ f) = (d \circ f) \bullet (f \circ d')$; (2) $(d \circ d') \bullet (f \circ f) = (f \circ f) \bullet (d \circ d')$.*

Proof. (1) Since d is an (r,l)-(f,g)-derivation and d' is an (l,r)-(f,g)-derivation of X, then for all $x, y \in X$, $(d \circ d')(x * y) = d((d'(x) * f(y)) \wedge (g(x) * d'(y))) = d(d'(x) * f(y)) = (f(d'(x)) * d(f(y))) \wedge (d(d'(x)) * g(f(y))) = (f \circ d'(x)) * (d \circ f(y))$. Also, d is a (l,r)-(f,g)-derivation and d' is a (r,l)-(f,g)-derivation of X. Hence, for all $x, y \in X$, $(d \circ d')(x * y) = d((f(x) * d'(y)) \wedge (d'(x) * g(y))) = d(f(x) * d'(y)) = (d(f(x)) * f(d'(y))) \wedge (g(f(x)) * d(d'(y))) = (d \circ f(x)) * (f \circ d'(y))$. Now, we obtain $(f \circ d'(x)) * (d \circ f(y)) = (d \circ f(x)) * (f \circ d'(y))$, for all $x, y \in X$. By putting $x = y$, we have $(f \circ d'(x)) * (d \circ f(x)) = (d \circ f(x)) * (f \circ d'(x))$. So $(f \circ d' \bullet d \circ f)(x) = (d \circ f \bullet f \circ d')(x)$, for all $x \in X$.

(2) The proof is similar to the proof of (1). ■

Let $Der(X)$ denotes the set of all (f,g)-derivations on X. Let $d, d' \in Der(X)$. Define the binary operation $\wedge$ as follows: $(d \wedge d')(x) = d(x) \wedge d'(x)$, for all $x \in X$.

THEOREM 12. *If X is a commutative B-algebra, then $(Der(X), \wedge)$ is a semigroup.*

Proof. Suppose that d, d' are (l,r)-(f,g)-derivations of X. We prove $d \wedge d'$ is also an (l,r)-(f,g)-derivation. For all $x, y \in X$, $(d \wedge d')(x * y) = d(x * y) \wedge d'(x * y) = d(x * y) = d(x) * f(y) = (d(x) \wedge d'(x)) * f(y) = (d \wedge d')(x) * f(y) = ((d \wedge d')(x) * f(y)) \wedge (g(x) * (d \wedge d')(y))$. So, $d \wedge d'$ is a (l,r)-(f,g)-derivation of X.

Now, suppose that d, d' are (r,l)-(f,g)-derivations of X. Then, for all $x, y \in X$, $(d \wedge d')(x * y) = d(x * y) \wedge d'(x * y) = d(x * y) = f(x) * d(y) = f(x) * (d(y) \wedge d'(y)) = f(x) * (d \wedge d')(y) = (f(x) * (d \wedge d')(y)) * ((d \wedge d')(x) * g(y))$. So, $d \wedge d'$ is an (r,l)-(f,g)-derivation of X. Therefore, $d \wedge d' \in Der(X)$. Let $d, d', d'' \in Der(X)$. We prove $d \wedge (d' \wedge d'') = (d \wedge d') \wedge d''$ (associative property). If $x, y \in X$, then $(d \wedge (d' \wedge d''))(x * y) = d(x * y) \wedge (d' \wedge d'')(x * y) = d(x * y)$. Also, we have $((d \wedge d') \wedge d'')(x * y) = (d \wedge d')(x * y) = d(x * y) \wedge d'(x * y) = d(x * y)$. This shows that $(d \wedge (d' \wedge d''))(x * y) = ((d \wedge d') \wedge d'')(x * y)$, for all $x, y \in X$. By putting $y = 0$, we obtain $d \wedge (d' \wedge d'') = (d \wedge d') \wedge d''$. Therefore, $(Der(X), \wedge)$ is a semigroup. ■

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