

## SOME REMARKS ON SEQUENCE SELECTION PROPERTIES USING IDEALS

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**Abstract.** In this paper we follow the line of recent works of Das and his co-authors where certain results on open covers and selection principles were studied by using the notion of ideals and ideal convergence, which automatically extend similar classical results (where finite sets are used). Here we further introduce the notions of  $\mathcal{I}$ -Sequence Selection Property ( $\mathcal{I}$ -SSP),  $\mathcal{I}$ -Monotonic Sequence Selection Property ( $\mathcal{I}$ -MSSP) of  $C_p(X)$  which extend the notions of Sequence Selection Property and Monotonic Sequence Selection Property of  $C_p(X)$  respectively. We then make certain observations on these new types of SSP in terms of  $\mathcal{I}$ - $\gamma$ -covers.

### 1. Introduction

To recall a brief history of this line of investigation we start by mentioning that the notion of ideal convergence was used under the name of filter convergence in topology and set theory since its introduction by H. Cartan in 1937 [5,6]. The nature or asymptotic density is defined as follows: If  $\mathbb{N}$  denotes the set of natural numbers and  $K \subset \mathbb{N}$  then  $K_n$  denotes the set  $\{k \in K : k \leq n\}$  and  $|K_n|$  stands for the cardinality of the set  $K_n$ . The asymptotic density of the subset  $K$  is defined by

$$d(K) = \lim_{n \rightarrow \infty} \frac{|K_n|}{n}$$

provided the limit exists.

Using this idea of asymptotic density, the idea of convergence of a real sequence was extended to statistical convergence by H. Fast [11] as follows. A sequence  $\{x_n\}_{n \in \mathbb{N}}$  of points in a metric space  $(X, \rho)$  is said to be statistically convergent to  $\ell$  if for arbitrary  $\varepsilon > 0$ , the set  $K(\varepsilon) = \{k \in \mathbb{N} : d(x_k, \ell) \geq \varepsilon\}$  has asymptotic density zero. In [11], Fast proved the Steinhaus theorem about statistical convergence which was presented by H. Steinhaus in 1949. A lot of investigations have been done on this very interesting convergence and its topological consequences after the initial works by Fridy [12] and Šalat [16].

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On the other hand, in [14], an interesting generalization of the notion of statistical convergence was proposed. Namely it is easy to check that the family  $\mathcal{I}_d = \{A \subset \mathbb{N} : d(A) = 0\}$  forms a non-trivial admissible (or free) ideal of  $\mathbb{N}$  (A family  $\mathcal{I} \subset 2^Y$  of subsets of a nonempty set  $Y$  is said to be an ideal in  $Y$  if (i)  $A, B \in \mathcal{I}$  imply  $A \cup B \in \mathcal{I}$ , (ii)  $A \in \mathcal{I}, B \subset A$  imply  $B \in \mathcal{I}$ . Here we consider an ideal of  $\mathbb{N}$  and without any loss of generality we also assume that  $\bigcup_{A \in \mathcal{I}} A = \mathbb{N}$  which implies that  $\{k\} \in \mathcal{I}$  for each  $k \in \mathbb{N}$ . Such ideals were sometimes called admissible ideals in the literature [14] (which are also called free ideals). If  $\mathcal{I}$  is a proper ideal in  $Y$  (i.e.,  $Y \notin \mathcal{I}, \mathcal{I} \neq \{\emptyset\}$ ) then the family of sets  $\mathcal{F}(\mathcal{I}) = \{M \subset Y : \text{there exists } A \in \mathcal{I} : M = Y \setminus A\}$  is a filter in  $Y$ . It is called the filter associated with the ideal  $\mathcal{I}$ ). Thus one may consider an arbitrary ideal  $\mathcal{I}$  of  $\mathbb{N}$  and define  $\mathcal{I}$ -convergence of a sequence by replacing the sets of density zero by the members of the ideal. For different applications of ideals one can see [1, 7–9] where more references can be found. It should also be noted that these notions of convergence or for that matter any such notion involving ideals depend on the enumeration of pieces.

Of particular interest to our context is the investigation of open covers and selection principles using ideals recently done in [7,8].

As a natural consequence we continue our investigations of notions extended via ideals and introduce certain types of sequence selection properties namely  $\mathcal{I}$ -Sequence Selection Property ( $\mathcal{I}$ -SSP) and  $\mathcal{I}$ -Monotonic Sequence Selection Property ( $\mathcal{I}$ -MSSP) of  $C_p(X)$ . In this paper we observe several relationships between the  $\mathcal{I}$ -SSP ( $\mathcal{I}$ -MSSP) of  $C_p(X)$  and the selection principles related to the  $\mathcal{I}$ - $\gamma$ -covers of  $X$  in the context of ideals. For a perfectly normal space  $X$  we obtain an equivalent condition for which  $C_p(X)$  has the  $\mathcal{I}$ -SSP.

## 2. Basic definitions

Throughout the paper  $(X, \tau)$  stands for a Hausdorff topological space and  $\mathcal{I}$  stands for a proper admissible ideal of  $\mathbb{N}$ .

Recall that the usual definition of convergence of a sequence was extended by using an ideal in [14] as follows: A sequence  $\{x_n\}_{n \in \mathbb{N}}$  of real numbers is said to be  $\mathcal{I}$ -convergent to  $x \in \mathbb{R}$  if for each  $\varepsilon > 0$ , the set  $A(\varepsilon) = \{n \in \mathbb{N} : |x_n - x| \geq \varepsilon\} \in \mathcal{I}$ . Consequently the usual idea of pointwise convergence of a sequence of functions was extended via ideals in [14] (see also [1]) as follows. Let  $X$  be a nonempty set and let  $f_n, f$  be real valued functions defined on  $X$ . A sequence  $\{f_n\}_{n \in \mathbb{N}}$  of functions is said to be  $\mathcal{I}$ -pointwise convergent to  $f$  if for each  $x \in X$  and for each  $\varepsilon > 0$  there exists a set  $A = A(x, \varepsilon) \in \mathcal{I}$  such that  $n \in \mathbb{N} \setminus A$  implies  $|f_n(x) - f(x)| < \varepsilon$  and in this case we write  $f_n \xrightarrow{\mathcal{I}} f$  (or  $\mathcal{I}\text{-}\lim_{n \rightarrow \infty} f_n = f$ ).

Recall that a collection  $\mathcal{U}$  of subsets of  $X$  is a cover of  $X$  if  $X \notin \mathcal{U}$  and  $X = \bigcup \mathcal{U}$ . Following [8] we will consider the following definitions. A countable open cover  $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$  of  $X$  is said to be an  $\mathcal{I}$ - $\gamma$ -cover if for each  $x \in X$  the set  $\{n \in \mathbb{N} : x \notin U_n\}$  belongs to  $\mathcal{I}$ . The set of all  $\mathcal{I}$ - $\gamma$ -covers will be denoted by  $\mathcal{I}\text{-}\Gamma$ . We say that a topological space  $X$  has  $\mathcal{I}$ -Hurewicz property if for each sequence  $(\mathcal{U}_n : n \in \mathbb{N})$  of open covers of  $X$  there are finite  $\mathcal{V}_n \subset \mathcal{U}_n, n \in \mathbb{N}$  such that for each

$x \in X$ ,  $\{n \in \mathbb{N} : x \notin \bigcup \mathcal{V}_n\} \in \mathcal{I}$ . Several investigations related to these properties can be found in [7–9].

For  $\mathcal{I} = \mathcal{I}_{fin}$  the ideal of finite subsets of  $\mathbb{N}$  we get the standard notions of  $\gamma$ -cover and Hurewicz property. For applications involving these classical covering properties related to the sequence of functions one can consult [2,3]. But unlike the classical definition these definitions depend on the enumeration of pieces. An  $\mathcal{I}$ - $\gamma$ -cover may not remain an  $\mathcal{I}$ - $\gamma$ -cover for a changed enumeration. The enumeration can be changed only under special circumstances. For example recall that two ideals  $\mathcal{I}$  and  $\mathcal{J}$  are called *Rudin-Keisler equivalent* if there is a bijection  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that

$$A \in \mathcal{I} \iff f(A) \in \mathcal{J}.$$

Clearly if the enumeration of an  $\mathcal{I}$ - $\gamma$ -cover is changed by this bijection  $f$  then it becomes a  $\mathcal{J}$ - $\gamma$ -cover.

We now introduce our main definitions.

**DEFINITION 2.1.** Let  $X$  be a topological space. We say that  $X$  satisfies  $S_1(\mathcal{I}\text{-}\Gamma, \mathcal{I}\text{-}\Gamma)$  if for any sequence  $(\mathcal{U}_n : n \in \mathbb{N})$  of  $\mathcal{I}$ - $\gamma$ -covers of  $X$ , there exists a member of  $\mathcal{U}_n$  for each  $n \in \mathbb{N}$  which we denote by  $U_n$  such that the sequence  $\{U_n : n \in \mathbb{N}\}$  written in the enumeration of  $(\mathcal{U}_n : n \in \mathbb{N})$  is an  $\mathcal{I}$ - $\gamma$ -cover of  $X$ . Such a space will be called a  $S_1(\mathcal{I}\text{-}\Gamma, \mathcal{I}\text{-}\Gamma)$ -space.

Evidently any  $\gamma$ -space is a  $S_1(\mathcal{I}\text{-}\Gamma, \mathcal{I}\text{-}\Gamma)$ -space (see [4]). For  $\mathcal{I} = \mathcal{I}_d$ ,  $S_1(\mathcal{I}_d\text{-}\Gamma, \mathcal{I}_d\text{-}\Gamma)$  was considered by Di Maio and Kočinac in [10].

**DEFINITION 2.2.** Let  $X$  be a topological space. We say that  $C_p(X)$  has the  $\mathcal{I}$ -Sequence Selection Property (shortly,  $\mathcal{I}$ -SSP) if for any double sequence  $\{f_{n,m} : n, m \in \mathbb{N}\} \subseteq C_p(X)$  such that  $\mathcal{I}\text{-}\lim_{m \rightarrow \infty} f_{n,m} = 0$  for any  $n$  there are  $m_n$  such that  $\mathcal{I}\text{-}\lim_{n \rightarrow \infty} f_{n,m_n} = 0$ .

**DEFINITION 2.3.** Let  $X$  be a topological space. We say that  $C_p(X)$  has the  $\mathcal{I}$ -Monotonic Sequence Selection Property (shortly,  $\mathcal{I}$ -MSSP) if for any double sequence  $\{f_{n,m} : n, m \in \mathbb{N}\} \subseteq C_p(X)$  such that  $\{f_{m,n} : m \in \mathbb{N}\}$  is monotonically decreasing and  $\lim_{m \rightarrow \infty} f_{n,m} = 0$  for any  $n$  there are  $m_n$  such that  $\mathcal{I}\text{-}\lim_{n \rightarrow \infty} f_{n,m_n} = 0$ .

Note that  $\mathcal{I}_{fin}$ -SSP ( $\mathcal{I}_{fin}$ -MSSP) of  $C_p(X)$  corresponds to the Sequence Selection Property (Monotonic Sequence Selection Property) of  $C_p(X)$  [17].

**DEFINITION 2.4.** A cover  $\{V_n : n \in \mathbb{N}\}$  is said to be a refinement of the cover  $\{U_n : n \in \mathbb{N}\}$  if  $V_n \subset U_n$  for each  $n \in \mathbb{N}$ . We say that  $X$  is an  $\mathcal{I}$ - $\gamma\gamma_{co}$ -space if for every  $\mathcal{I}$ - $\gamma$ -cover  $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$  of  $X$  there exists a refinement  $\mathcal{V} = \{V_n : n \in \mathbb{N}\}$  of  $\mathcal{U}$  such that  $\mathcal{V}$  (with the same enumeration) is an  $\mathcal{I}$ - $\gamma$ -cover consisting of clopen sets.

It should be noted that for a perfectly normal space  $X$  with  $\text{Ind}(X) = 0$ , the notion of  $\mathcal{I}_{fin}$ - $\gamma\gamma_{co}$ -space corresponds to the  $(\gamma, \gamma)$ -shrinkable space as introduced by M. Sakai in [15] (see also [13]).

### 3. Main results

**THEOREM 3.1.** *If  $X$  satisfies  $S_1(\mathcal{I}\text{-}\Gamma, \mathcal{I}\text{-}\Gamma)$  then  $C_p(X)$  has  $\mathcal{I}$ -SSP.*

*Proof.* Let  $\{f_{n,m} : n, m \in \mathbb{N}\} \subseteq C_p(X)$  be such that  $\mathcal{I}\text{-}\lim_{m \rightarrow \infty} f_{n,m} = 0$  for all  $n \in \mathbb{N}$ . Assume that  $X$  is infinite and choose a sequence  $\{x_m : m \in \mathbb{N}\}$  of distinct points of  $X$ . Fix  $n$  and for each  $m \in \mathbb{N}$ , define  $U_m^{(n)} = \{x \in X : |f_{n,m}(x)| < \frac{1}{2^n} \wedge x \neq x_m\}$ . Choose  $n \in \mathbb{N}$  and  $x \in X$ . If  $x = x_k$  for some  $k$ , then  $\{m : x_k \notin U_m^{(n)}\} = \{m : |f_{n,m}(x_k)| \geq \frac{1}{2^n}\} \cup \{k\} \in \mathcal{I}$ . Also if  $x \neq x_k$  for any  $k$ , then  $\{m : x \notin U_m^{(n)}\} = \{m : |f_{n,m}(x)| \geq \frac{1}{2^n}\} \in \mathcal{I}$  as  $\mathcal{I}\text{-}\lim_{m \rightarrow \infty} f_{n,m} = 0$ . We have for each  $n$ ,  $\mathcal{U}_n = \{U_m^{(n)} : m \in \mathbb{N}\}$  is an  $\mathcal{I}$ - $\gamma$ -cover of  $X$ . Now apply  $S_1(\mathcal{I}\text{-}\Gamma, \mathcal{I}\text{-}\Gamma)$  to the sequence  $(\mathcal{U}_n : n \in \mathbb{N})$  to obtain  $U_{m_n}^{(n)} \in \mathcal{U}_n$  for each  $n \in \mathbb{N}$  such that the corresponding sequence  $\{U_{m_n}^{(n)} : n \in \mathbb{N}\}$  is an  $\mathcal{I}$ - $\gamma$ -cover of  $X$ . Choose  $\varepsilon > 0$ . Then there is a  $n_0 \in \mathbb{N}$  such that  $\frac{1}{2^{n_0}} < \varepsilon$ . Now choose  $x \in X$ . It follows that  $\{n \geq n_0 : |f_{n,m_n}(x)| > \varepsilon\} \subseteq \{n \geq n_0 : |f_{n,m_n}(x)| \geq \frac{1}{2^n}\} \subseteq \{n \geq n_0 : x \notin U_{m_n}^{(n)}\} \in \mathcal{I}$  as  $\{U_{m_n}^{(n)} : n \in \mathbb{N}\}$  is an  $\mathcal{I}$ - $\gamma$ -cover of  $X$ . Consequently  $\{n \in \mathbb{N} : |f_{n,m_n}(x)| > \varepsilon\} \in \mathcal{I}$  and hence  $\mathcal{I}\text{-}\lim_{n \rightarrow \infty} f_{n,m_n} = 0$  as required. ■

**THEOREM 3.2.** *Let  $X$  be a perfectly normal topological space with the property  $\mathcal{I}\text{-}\gamma\text{co}$ . If  $C_p(X)$  has  $\mathcal{I}$ -SSP, then  $X$  is a  $S_1(\mathcal{I}\text{-}\Gamma, \mathcal{I}\text{-}\Gamma)$ -space.*

*Proof.* Let  $(\mathcal{U}_n : n \in \mathbb{N})$  be a sequence of  $\mathcal{I}$ - $\gamma$ -covers of  $X$  where  $\mathcal{U}_n = \{U_{n,m} : m \in \mathbb{N}\}$ . Take the corresponding sequence of clopen refinements  $(\mathcal{V}_n : n \in \mathbb{N})$  and enumerate bijectively as  $\mathcal{V}_n = \{V_{n,m} : m \in \mathbb{N}\}$  (note that  $V_{n,m} \subset U_{n,m}, \forall n, m \in \mathbb{N}$ ). Define for all  $n, m$ , continuous functions  $f_{n,m} : X \rightarrow \mathbb{I}$  as characteristic functions of  $X \setminus V_{n,m}$  (where  $\mathbb{I}$  is the unit interval). Since  $\mathcal{V}_n$  is an  $\mathcal{I}$ - $\gamma$ -cover so  $\{m \in \mathbb{N} : x \notin V_{n,m}\} \in \mathcal{I}$ . Choose  $\varepsilon > 0$ . Now for each  $x \in X$ , the set  $A(x, \varepsilon) = \{m \in \mathbb{N} : f_{n,m}(x) > \varepsilon\} = \{m \in \mathbb{N} : f_{n,m}(x) = 1\} = \{m \in \mathbb{N} : x \notin V_{n,m}\} \in \mathcal{I}$  if  $\varepsilon < 1$  and if  $\varepsilon \geq 1$ ,  $A(x, \varepsilon) = \emptyset \in \mathcal{I}$ , i.e.,  $\mathcal{I}\text{-}\lim_{m \rightarrow \infty} f_{n,m} = 0$ . Hence choose  $\{m_n : n \in \mathbb{N}\} \subset \mathbb{N}$  such that  $\mathcal{I}\text{-}\lim_{n \rightarrow \infty} f_{n,m_n} = 0$ . For each  $x \in X$  we write  $B(x) = \{n \in \mathbb{N} : f_{n,m_n}(x) < 1\}$ . Clearly  $\mathbb{N} \setminus B(x) \in \mathcal{I}$ . Again we have  $B(x) = \{n \in \mathbb{N} : x \in V_{n,m_n}\}$  i.e.  $\{n \in \mathbb{N} : x \notin V_{n,m_n}\} = \mathbb{N} \setminus B(x) \in \mathcal{I}$  which implies that  $\{V_{n,m_n} : n \in \mathbb{N}\}$  is an  $\mathcal{I}$ - $\gamma$ -cover of  $X$ . For each  $n$  writing  $U_n = U_{n,m_n} \supset V_{n,m_n}$  we observe that  $C(x) = \{n \in \mathbb{N} : x \notin U_n\} \subset \{n \in \mathbb{N} : x \notin V_{n,m_n}\} \in \mathcal{I}$  i.e.  $C(x) \in \mathcal{I}$ . This proves that  $\{U_n : n \in \mathbb{N}\}$  is the required  $\mathcal{I}$ - $\gamma$ -cover of  $X$ . ■

Combining Theorem 3.1 and Theorem 3.2 we have the following

**THEOREM 3.3.** *Let  $X$  be a perfectly normal topological space with the property  $\mathcal{I}\text{-}\gamma\text{co}$ . Then the following conditions are equivalent.*

- (1)  $X$  is a  $S_1(\mathcal{I}\text{-}\Gamma, \mathcal{I}\text{-}\Gamma)$ -space.
- (2)  $C_p(X)$  has  $\mathcal{I}$ -SSP.

We obtain the following result related to the the  $\mathcal{I}$ -Monotonic Sequence Selection Property of  $C_p(X)$ .

**THEOREM 3.4.** *Let  $X$  be a topological space with the  $\mathcal{I}$ -Hurewicz property. Then  $C_p(X)$  has  $\mathcal{I}$ -MSSP.*

*Proof.* For each  $n$ , let  $\{f_{n,m} : m \in \mathbb{N}\}$  be a sequence in  $C_p(X)$  which is pointwise monotonically decreasing and  $\lim_{m \rightarrow \infty} f_{n,m} = 0$ . Assume that  $X$  is infinite and choose a sequence  $\{x_m : m \in \mathbb{N}\}$  of distinct points of  $X$ . Fix  $n$  and for each  $m \in \mathbb{N}$ , define  $U_m^{(n)} = \{x \in X : |f_{n,m}(x)| < \frac{1}{2^n} \wedge x \neq x_m\}$ . Since each  $f_{n,m}$  is continuous, each  $U_m^{(n)}$  is an open subset of  $X$ . Define for each  $n$ ,  $\mathcal{U}_n = \{U_m^{(n)} : m \in \mathbb{N}\}$ . Since  $\lim_{m \rightarrow \infty} f_{n,m} = 0$ ,  $\mathcal{U}_n$  is an open cover of  $X$ . Since this sequence is pointwise monotonically decreasing, it follows that  $U_m^{(n)} \subseteq U_k^{(n)}$  whenever  $m \leq k$ . Also it follows (from the definition of  $U_m^{(n)}$ 's) that for each  $n$ ,  $\mathcal{U}_n$  is an  $\mathcal{I}$ - $\gamma$ -cover of  $X$ . Apply the  $\mathcal{I}$ -Hurewicz property to the sequence  $(\mathcal{U}_n : n \in \mathbb{N})$  to find for each  $n$ , a  $m_n$  such that for each  $x \in X$ , the set  $\{n \in \mathbb{N} : x \notin U_{m_n}^{(n)}\} \in \mathcal{I}$ . Choose  $\varepsilon > 0$ . Then there is a  $n_0 \in \mathbb{N}$  such that  $\frac{1}{2^{n_0}} < \varepsilon$ . Clearly for each  $x \in X$ , the set  $\{n \geq n_0 : |f_{n,m_n}(x)| > \varepsilon\} \subseteq \{n \geq n_0 : x \notin U_{m_n}^{(n)}\} \in \mathcal{I}$ . Consequently  $\{n \in \mathbb{N} : |f_{n,m_n}(x)| > \varepsilon\} \in \mathcal{I}$  and hence  $\mathcal{I}\text{-}\lim_{n \rightarrow \infty} f_{n,m_n} = 0$  as required. ■

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