МАТЕМАТИЧКИ ВЕСНИК
46 (1994), 17–23

UDC 515.122.8
оригинални научни рад
research paper

EXTREMAL DISCONNECTEDNESS IN FUZZY TOPOLOGICAL SPACES

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Abstract. The notion of extremal disconnectedness is introduced for fuzzy topological spaces. Various properties of this notion are discussed. Interrelationship of some weaker forms of fuzzy continuity is studied in the light of extremal diconnectedness.

1. Introduction

Connectedness and its related notions are studied by various researchers for fuzzy topological space ([3], [5], [13]). In many cases, the study while remaining parallel to its counterpart in general topology shows divergence due to the fuzzy setting. For example, there are four types of connectedness, namely c_i-connectedness ($i = 1, 2, 3, 4$) defined for an arbitrary fuzzy set. But when considered globally, that is, when the fuzzy set is considered to be the whole space, all these four types coincide to one type. Because of these diversities, study of connectedness and related concepts remain to be an interesting one in the area of fuzzy topology. In this paper, we add to the existing literature one more notion, namely, extremal disconnectedness. Various properties of extremal disconnectedness is studied and the notion is shown to be compatible with the existing fuzzy topological notions such as T_2-axiom, fuzzy regularity, c-connectedness, zero-dimension etc. Also following localized approach, we provide a number of characterizations of fuzzy weak continuity. Alongwith, we study the interrelationship between fuzzy weak continuity and fuzzy semi-continuity in the context of extremal diconnectedness.

2. Preliminaries

Throughout the paper, X, Y, Z etc. denote ordinary sets while μ, η, σ etc. denote fuzzy sets defined on an arbitrary set. The fuzzy sets are defined with respect to the closed unit interval $I = [0, 1]$. The union and intersection of a family of fuzzy sets $\{\mu_i\}$ are denoted $\vee\mu_i$ and $\wedge\mu_i$ respectively. The constant fuzzy sets which take each member of X to zero and one respectively are denoted by 0_X and

Keywords: Fuzzy point, neighbourhood, quasi-neighbourhood, quasi-coincident fuzzy sets, fuzzy continuity

1_X respectively. A fuzzy point with support x and value α, $0 < \alpha \leqslant 1$, is denoted by x_α. While $x_\alpha \leqslant \mu$ implies $\alpha \leqslant \mu(x)$; $x_\alpha \,\varepsilon\, \mu$ implies $\alpha < \mu(x)$ and $\mu \leqslant \eta$ implies $\mu(x) \leqslant \eta(x)$ for each x. Two fuzzy sets μ and η are said to overlap, denoted by $\mu \, q \, \eta$, if there exists x in X such that $\mu(x) + \eta(x) > 1$. For the definition of a fuzzy topological space (an *fts* in brief), we refer to Chang [10]. For other definitions, results and notations used in the paper, see [1], [6], [13] or [16].

3. Extremal disconnectedness

DEFINITION 3.1. An *fts* X is called *extremally disconnected* if for every open fuzzy set μ in X, $\mathrm{cl}(\mu)$ is a crisp clo-open fuzzy set in X.

EXAMPLE 3.2. Let X be a non-empty set and $T = \{0_X, 1_X\} \cup \{\mu_\alpha\}_{\alpha \in \Lambda}$, where μ_α is any fuzzy set defined on X such that $\frac{1}{2} < \mu_\alpha(x) < 1$, for every x in X. Then the *fts* $\langle X, T \rangle$ is extremally disconnected, although X is not c-connected.

Below we give a necessary and sufficient condition for an *fts* to be extremally disconnected.

THEOREM 3.3. *An fts X is extremally disconnected iff for every pair of non-overlaping open fuzzy sets μ and η in X, $\mathrm{cl}(\mu)$ and $\mathrm{cl}(\eta)$ are non-overlaping crisp fuzzy sets in X.*

Proof. Let X be an extremally disconnected *fts* and μ and η be open fuzzy sets in X such that $\mu \,\not{q}\, \eta$. Suppose, if possible, $\mathrm{cl}(\mu)$ overlaps with $\mathrm{cl}(\eta)$. Then $(\mathrm{cl}(\mu))(y) + (\mathrm{cl}(\eta))(y) > 1$, for some y in X. Then the fuzzy point $\mathcal{Y}_{(\mathrm{cl}(\eta))(y)} \leqslant \mathrm{cl}(\eta)$ and $\mathcal{Y}_{(\mathrm{cl}(\eta))(y)} \; q \; \mathrm{cl}(\mu)$. X being extremally disconnected, $\mathrm{cl}(\mu)$ is open. Then $\mathrm{cl}(\mu)$ is a Q-nbd of $\mathcal{Y}_{(\mathrm{cl}(\eta))(y)}$ and $\mathcal{Y}_{(\mathrm{cl}(\eta))(y)} \leqslant \mathrm{cl}(\eta)$. Therefore $\eta \; q \; \mathrm{cl}(\mu)$. Let $\eta(x) + (\mathrm{cl}(\mu))(x) > 1$. This implies that $x_{1-\eta(x)}$ properly belongs to $\mathrm{cl}(\mu)$, or, $x_{1-\eta(x)+\varepsilon} \leqslant \mathrm{cl}(\mu)$ for some positive real number ε. Also $x_{1-\eta(x)+\varepsilon} \; q \; \eta$. Therefore $\mu \, q \, \eta$, which is a contradiction.

Conversely, let μ be an open fuzzy set in X. Then μ and $1_X - \mathrm{cl}(\mu)$ are two open fuzzy sets such that $\mu \,\not{q}\, 1_X - \mathrm{cl}(\mu)$. Therefore $\mathrm{cl}(\mu) \,\not{q}\, \mathrm{cl}(1_X - \mathrm{cl}(\mu))$ and $\mathrm{cl}(\mu)$ is a crisp fuzzy set in X. Now, $\mathrm{cl}(\mu) \,\not{q}\, \mathrm{cl}(1_X - \mathrm{cl}(\mu))$ implies that $\mathrm{cl}(\mu) \leqslant 1_X - \mathrm{cl}(1_X - \mathrm{cl}(\mu)) = 1_X - (1_X - \mathrm{int}(\mathrm{cl}(\mu))) = \mathrm{int}(\mathrm{cl}(\mu))$; so that $\mathrm{cl}(\mu) = \mathrm{int}(\mathrm{cl}(\mu))$. Thus $\mathrm{cl}(\mu)$ is a crisp clo-open fuzzy set in X. Hence X is extremally disconnected. This completes the proof. ∎

THEOREM 3.4. *Every open subspace of an extremally disconnected fts is extremally disconnected.*

Proof. Easy.

In [3] and [5], c-disconnectedness for general fuzzy topological spaces is introduced and studied. An *fts* X is *c-disconnected* if there exist two proper open fuzzy sets μ and η such that $\mu \wedge \eta = 0_X$ and $\mu \vee \eta = 1_X$. It follows that an *fts* X is c-disconnected iff there exists a proper crisp clo-open fuzzy set in X. The following example alongwith Example 3.2 shows that c-disconnectedness and extremal disconnectedness are two independent notions.

EXAMPLE 3.5. Let A and B be non-empty disjoint sets and $X = A \cup B$. Let μ_A^α and η_B^β be fuzzy sets defined by

$$\mu_A^\alpha(x) = \begin{cases} \alpha, & \text{for } x \in A,\ 0 \leqslant \alpha \leqslant 1, \\ 0, & \text{otherwise,} \end{cases} \qquad \eta_B^\beta(y) = \begin{cases} \beta, & \text{for } y \in B,\ 0 \leqslant \beta \leqslant 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $T(x) = \{\mu_A^\alpha \vee \eta_B^\beta\}_{0 \leqslant \alpha, \beta \leqslant 1}$. Then the *fts* $\langle X, T(x)\rangle$ is c-disconnected and μ_A^1 and η_B^1 are two non-zero open fuzzy sets in X which are providing a c-disconnectedness of 1_X. But this *fts* fails to be extremally disconnected as the closure of the open fuzzy set $\mu_A^{1/2} \vee \eta_B^{1/2}$ is not crisp.

The following theorem establishes relationship between c-disconnectedness and extremal disconnectedness.

THEOREM 3.6. *Every subspace of an extremally disconnected Hausdorff fts is c-disconnected.*

Proof. Let X be an extremally disconnected Hausdorff *fts* and A be a subspace of X. Let x_α $(0 < \alpha < 1)$ and y_β $(0 < \beta < 1)$ be two fuzzy points in A with distinct supports. As X is T_2, considering x_α and y_β as fuzzy points in X, we get two open fuzzy sets μ and η in X such that $x_\alpha\ q\ \eta$, $y_\beta\ q\ \eta$ and $\mu \wedge \eta = 0_X$. This implies that $x_{\alpha'}\ \varepsilon\ \mu$, $y_{\beta'}\ \varepsilon\ \eta$ and $\mu \wedge \eta = 0_X$, where $\alpha' = 1 - \alpha$, $\beta' = 1 - \beta$. Now, X being extremally disconnected, $\mathrm{cl}(\mu)$ and $\mathrm{cl}(\eta)$ are crisp clo-open fuzzy sets in X, so that $(\mathrm{cl}(\mu))^a$ and $(\mathrm{cl}(\eta))^a$, where $(\mathrm{cl}(\mu))^a = \mathrm{cl}(\mu)|_A$, are crisp clo-open fuzzy sets in A. Also $x_{\alpha'}\ \varepsilon\ \mu$ and x is in A imply that $x_{\alpha'}\ \varepsilon\ (\mathrm{cl}(\mu))^a$. Thus $(\mathrm{cl}(\mu))^a \neq 0_A$. Similarly $y_{\beta'}\ \varepsilon\ (\mathrm{cl}(\eta))^a$. Again that $\mu \wedge \eta = 0_X$ and X is extremally disconnected together imply that $\mathrm{cl}(\mu) + \mathrm{cl}(\eta) \leqslant 1_X$, in view of Theorem 3.3; whence $(\mathrm{cl}(\mu))^a + (\mathrm{cl}(\eta))^a \leqslant 1_A$. Therefore $(\mathrm{cl}(\mu))^a(y) + (\mathrm{cl}(\eta))^a(y) \leqslant 1$, that is, $(\mathrm{cl}(\mu))^a(y) + 1 - \beta < (\mathrm{cl}(\mu))^a(y) + (\mathrm{cl}(\eta))^a(y) \leqslant 1$. Or, in other words $(\mathrm{cl}(\mu))^a(y) < \beta$. Thus $(\mathrm{cl}(\mu))^a \neq 1_A$. Therefore $(\mathrm{cl}(\mu))^a$ is a proper crisp clo-open fuzzy set in A. Consequently A is c-disconnected. This completes the proof. ■

REMARK. In the above theorem, the condition that "X is Hausdorff" can not be dropped. For example, the *fts* given in Example 3.2 is extremally disconnected. But here even X itself is not c-disconnected. It may be noticed that X is not a Hausdorff *fts* in this case.

EXAMPLE 3.7. Let $X = \{x, y\}$, $S = \{\mu_1, \mu_2\} \cup \{\mu_{\alpha\beta}\}_{1/2 < \alpha, \beta \leqslant 1}$ and consider the fuzzy topology $T(x)$ generated by S, where

$$\begin{aligned} &\mu_1(x) = 1, \quad \mu_2(x) = 0, \quad \mu_{\alpha\beta}(x) = \alpha, \\ &\mu_1(y) = 0, \quad \mu_2(y) = 1, \quad \mu_{\alpha\beta}(y) = \beta. \end{aligned}$$

Then $\langle X, T(x)\rangle$ is both extremally disconnected as well as c-disconnected. Here it can be verified that X is a T_2-*fts* .

DEFINITION 3.8.[8] An *fts* X is called *zero-dimensional* if every crisp fuzzy point in X has a base of crisp clo-open fuzzy sets.

An *fts* X is called a *regular fuzzy space* ([1], [12]) if each open fuzzy set μ in X is a union of open fuzzy sets μ_i in X, such that $\mathrm{cl}(\mu_i) \leqslant \mu$ for each i. We have

THEOREM 3.9. *Every non-empty extremally disconnected regular fts is zero-dimensional.*

Proof. Easy.

In the rest of the paper, we discuss extremal disconnectedness in the light of some weaker forms of fuzzy continuity.

DEFINITION 3.10.[4] A mapping $f\colon X \to Y$ from an *fts* X to an *fts* Y is said to be *fuzzy weakly continuous* (briefly f.w.c.) at a fuzzy point x_α, if for each neighbourhood η of $f(x_\alpha)$ in Y, there exists a neighbourhood μ of x_α in X such that $f(\mu) \leqslant \mathrm{cl}(\eta)$.

f is said to be *fuzzy weakly continuous* if it is so at each fuzzy point in X.

Below we provide several characterizations of fuzzy weak continuity. Its proof is similar to that of Theorem 3.4 [4] and it is left for the reader as an exercise.

THEOREM 3.11. *For a mapping $f\colon X \to Y$ from an fts X to an fts Y, the following are equivalent:*

(i) f is fuzzy weakly continuous;

(ii) for each open fuzzy set η in Y, $f^{-1}(\eta) \leqslant \mathrm{int}(f^{-1}(\mathrm{cl}(\eta)))$;

(iii) for each closed fuzzy set σ in Y, $\mathrm{cl}(f^{-1}(\mathrm{int}(\sigma))) \leqslant f^{-1}(\sigma)$;

(iv) f is f.w.c. at every x_α, where $0 < \alpha < 1$;

(v) for each fuzzy point x_α in X, and for every open fuzzy set η in Y properly containing $f(x_\alpha)$, there is an open fuzzy set μ properly containing x_α such that $f(\mu) \leqslant \mathrm{cl}(\eta)$;

(vi) for any fuzzy point x_α in X and for each Q-nbd η of $f(x_\alpha)$, $\mathrm{int}(f^{-1}(\mathrm{cl}(\eta)))$ is a Q-nbd of x_α;

(vii) for any fuzzy point x_α in X and for each Q-nbd η of $f(x_\alpha)$, there exists a Q-nbd μ of x_α such that $f(\mu) \leqslant \mathrm{cl}(\eta)$;

(viii) for each fuzzy net $\{S_n\}$ converging to any fuzzy point x_α, the image fuzzy net $\{f(S_n)\}$ eventually overlaps with the closure of each Q-nbd of $f(x_\alpha)$;

(ix) for each open fuzzy set η in Y, $\mathrm{cl}(f^{-1}(\eta)) \leqslant f^{-1}(\mathrm{cl}(\eta))$.

DEFINITION 3.12. A mapping $f\colon X \to Y$, where $A \subset X$, is said to be a *retraction of X onto A* if $f(x) = x$ iff x is in A.

THEOREM 3.13. *Let $A \subset X$ and $f\colon X \to A$ be a fuzzy weakly continuous retraction of X onto A. If X is extremally disconnected Hausdorff fts, then A is a closed subspace of X.*

Proof. Suppose, if possible, χ_A is not closed. Then there exists a fuzzy point x_α in X, such that $x_\alpha \leqslant \mathrm{cl}(\chi_A)$, but $x_\alpha \nleqslant \chi_A$. Then x is not in A. Therefore

$f(x) \neq x$. Since X is Hausdorff, there exist disjoint Q-nbds μ and η of x_α and $f(x_\alpha)$ respectively. Now, X being extremally disconnected, $\mu \wedge \mathrm{cl}(\eta) = 0_X$. For, if $y_\beta \leqslant \mu \wedge \mathrm{cl}(\eta)$, then $\mu(y) > 0$ and $(\mathrm{cl}(\eta))(y) > 0$. Since $\mathrm{cl}(\eta)$ is a crisp fuzzy set, $(\mathrm{cl}(\eta))(y) = 1$. Thus $y_1 \leqslant \mathrm{cl}(\eta)$ and $\mu(y) > 0$. Therefore, $y_1 \leqslant \mathrm{cl}(\eta)$ and $y_1 \; q \; \mu$. Hence $\mu \; q \; \eta$, which is a contradiction, as $\mu \wedge \eta = 0_X$.

Now, let σ be a Q-nbd of x_α. Then $\mu \wedge \sigma$ is a Q-nbd of x_α. Therefore $\mu \wedge \sigma$ overlaps with χ_A at some point z in X, as $x_\alpha \leqslant \mathrm{cl}(\chi_A)$. This implies that

$$(\mu \wedge \sigma)(z) \neq 0 \quad \text{and } z \text{ is in } A.$$

That is,

$$\mu(z) \neq 0, \quad \sigma(z) \neq 0 \quad \text{and } f(z) = z.$$

Therefore, $(\mathrm{cl}(\eta))(z) = 0$, $\sigma(z) \neq 0$ and $f(z) = z$, using the fact that $\mu \wedge \mathrm{cl}(\eta) = 0_X$. Thus,

$$(\mathrm{cl}(\eta))(f(z)) = 0, \quad f(\sigma)(f(z)) \neq 0.$$

Hence, $f(\sigma) \nleqslant \mathrm{cl}(\eta)$. This contradicts the fact that f is f.w.c. at x_α, in view of Theorem 3.11. Thus our assumption is wrong and consequently χ_A is a closed fuzzy set in X. Hence A is a closed subspace of X. ∎

Definition 3.14. (a) A fuzzy set μ is called *semi-open* if there exists an open fuzzy set μ_0 such that $\mu_0 \leqslant \mu \leqslant \mathrm{cl}(\mu_0)$.

(b) The complement of a semi-open fuzzy set is called a *semi-closed* fuzzy set.

(c) Intersection of all semi-closed fuzzy sets containing a fuzzy set μ is called the *semi-closure* of μ and is denoted by s-cl(μ).

Azad [9] has defined a fuzzy set σ to be semi-closed if there exists an open fuzzy set η in X such that

$$\mathrm{int}(1_X - \eta) \leqslant \sigma \leqslant 1_X - \eta.$$

However, this definition can easily be shown to be equivalent to the above mentioned definition.

For a fuzzy set μ, cl(μ) and s-cl(μ) need not be same. This can be verified from the following example.

Example 3.15. Let X be any non-empty set and $T(x) = \{0_X, \mu_{1/3}, \mu_{2/3}, 1_X\}$, where $\mu_i(x) = i$ for all x in X, $i = 1/3$, $2/3$. Then for the fuzzy set μ, defined by $\mu(x) = \frac{1}{2}$ for all x in X, we have s-cl$(\mu) = \mu$ and cl$(\mu) = \mu_{2/3}$.

Further, from this example it is clear that s-cl(μ) and cl(μ) need not be same even if μ is semi-open. However, if the *fts* is extremally disconnected, the closure and semi-closure of a semi-open fuzzy set turn out to be same. Before proving this we state the following result.

Theorem 3.16.[11] *Let μ be a fuzzy set in an fts X. Then $x_\alpha \leqslant$ s-*cl(μ) *iff every semi-open fuzzy set overlapping with x_α overlaps with μ.*

THEOREM 3.17. *In an extremally disconnected fts X, $\mathrm{cl}(\mu) = s\text{-}\mathrm{cl}(\mu)$ for every semi-open fuzzy set μ in X.*

Proof. In general, $\text{s-cl}(\mu) \leqslant \mathrm{cl}(\mu)$, as every closed fuzzy set is also semi-closed. Thus it suffices to show that $\mathrm{cl}(\mu) \leqslant \text{s-cl}(\mu)$ for every semi-open fuzzy set in X. Let x_α be a fuzzy point in X such that $x_\alpha \nleqslant \text{s-cl}(\mu)$. Then there exists, by Theorem 3.16, a semi-open fuzzy set η in X such that $x_\alpha\ q\ \eta$ and $\eta \not{q}\ \mu$. This implies that $x_\alpha\ q\ \eta$ and $\mathrm{int}(\eta) \not{q}\ \mathrm{int}(\mu)$. Since X is extremally disconnected, by Theorem 3.3, we have $\mathrm{cl}(\mathrm{int}(\mu)) \not{q}\ \mathrm{cl}(\mathrm{int}(\eta))$. Also by Theorem 4.2[9] , $\eta \leqslant \mathrm{cl}(\mathrm{int}(\eta))$. Therefore, $\mathrm{cl}(\mathrm{int}(\mu)) \not{q}\ \eta$. Again, μ being semi-open there exists an open fuzzy set μ_0 such that $\mu_0 \leqslant \mu \leqslant \mathrm{cl}(\mu_0)$. Therefore,

$$\mu_0 \leqslant \mathrm{int}(\mu) \leqslant \mu \leqslant \mathrm{cl}(\mu) \leqslant \mathrm{cl}(\mu_0).$$

This implies that

$$\mathrm{cl}(\mu_0) \leqslant \mathrm{cl}(\mathrm{int}(\mu)) \leqslant \mathrm{cl}(\mu) \leqslant \mathrm{cl}(\mathrm{cl}(\mu)) \leqslant \mathrm{cl}(\mathrm{cl}(\mu_0)).$$

Hence $\mathrm{cl}(\mu) = \mathrm{cl}(\mathrm{int}(\mu))$. Thus it follows that $\mathrm{cl}(\mu) \not{q}\ \eta$. Also $x_\alpha\ q\ \eta$. Therefore,

$$(\mathrm{cl}(\mu))(x) + \eta(x) \leqslant 1 \quad \text{and} \quad \alpha + \eta(x) > 1,$$

so that $\alpha > (\mathrm{cl}(\mu))(x)$. Hence $x_\alpha \nleqslant \mathrm{cl}(\mu)$. Consequently, $\mathrm{cl}(\mu) \leqslant \text{s-cl}(\mu)$. This proves the theorem. ■

The last theorem is in the spirit of Theorem 3.2 of Noiri [15]. It establishes the relationship between fuzzy semi-continuity and fuzzy weak continuity in the light of extremal disconnectedness. We first recall the definition of fuzzy semi-continuity.

DEFINITION 3.18.[7] A mapping $f\colon X \to Y$ from an *fts* X to *fts* Y is said to be *fuzzy semi-continuous* at a fuzzy point x_α if for every open fuzzy set η containing $f(x_\alpha)$ there exists a semi-open fuzzy set μ containing x_α in X such that $f(\mu) \leqslant \eta$.

f is said to be *fuzzy semi-comtinuous*, if it is so at every fuzzy point in X.

THEOREM 3.19. *A mapping $f\colon X \to Y$ from an fts X to an fts Y is fuzzy semi-continuous iff $f^{-1}(\eta)$ is semi-open for every open fuzzy set η in Y.*

Proof. Easy.

From the definition it is clear that fuzzy continuity implies fuzzy semi-continuity. The converse is, however, not true. Also, fuzzy weak continuity and fuzzy semi-continuity are two independent notions.

EXAMPLE 3.20. Let X be any non-empty set and $T_1(x) = \{0_X, \mu_{1/3}, 1_X\}$, $T_2(x) = \{0_X, \mu_{1/2}, 1_X\}$, where $\mu_j(x) = j$, $j = 1/3,\ 1/2$, for every x in X. Then the identity mapping $i\colon \langle X, T_1(x)\rangle \to \langle X, T_2(x)\rangle$ is fuzzy semi-continuous but neither fuzzy continuous nor fuzzy weakly continuous, as it is not f.w.c. at any x_α, where $1/3 < \alpha \leqslant 1/2$. Again the identity mapping $i'\colon \langle X, T_2(x)\rangle \to \langle X, T_1(x)\rangle$ is fuzzy weakly continuous but not fuzzy semi-continuous as it is not fuzzy semi-continuous at any x_α, where $0 < \alpha \leqslant 1/3$.

We are now in a position to prove the following theorem.

THEOREM 3.21. *If an fts X is extremally disconnected and $f\colon X \to Y$ is fuzzy semi-continuous, then f is fuzzy weakly continuous.*

Proof. Let η be an open fuzzy set in Y. Then $f^{-1}(\eta)$ is a semi-open fuzzy set in X, as f is fuzzy semi-continuous. Let $x_\alpha \nleqslant f^{-1}(\mathrm{cl}(\eta))$. Then $f(x_\alpha) \nleqslant \mathrm{cl}(\eta)$. Therefore there exists a closed fuzzy set σ containing η such that $f(x_\alpha) \nleqslant \sigma$. Since f is fuzzy semi-continuous, $f^{-1}(1_Y - \sigma)$ is semi-open in X. But

$$f^{-1}(1_Y - \sigma) = 1_X - f^{-1}(\sigma).$$

Thus $f^{-1}(\sigma)$ is a semi-closed fuzzy set in X. Also, $f^{-1}(\eta) \leqslant f^{-1}(\sigma)$ as $\eta \leqslant \sigma$ and $x_\alpha \nleqslant f^{-1}(\sigma)$ as $f(x_\alpha) \nleqslant \sigma$. This implies that $x_\alpha \nleqslant \text{s-cl}(f^{-1}(\eta))$. Therefore, by Theorem 3.17, $x_\alpha \nleqslant \mathrm{cl}(f^{-1}(\eta))$, as X is extremally disconnected. Thus $\mathrm{cl}(f^{-1}(\eta)) \leqslant f^{-1}(\mathrm{cl}(\eta))$. Hence by Theorem 3.11, f is fuzzy weakly continuous. ■

Here the condition "extremal disconnectedness" cannot be dropped. For example, $\langle X, T_1(x)\rangle$ is not extremally disconnected in Example 3.20. Although the identity mapping is fuzzy semi-continuous, it is not fuzzy weakly continuous.

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(received 28.07.1993.)

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